DO NON-STRICTLY STABLE LAWS ON POSITIVELY GRADUATED SIMPLY CONNECTED NILPOTENT LIE GROUPS LIE IN THEIR OWN DOMAIN OF NORMAL ATTRACTION?

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Abstract: In the classical case of the real line, it is clear from the very definition that non-degenerate stable laws always belong to their own domain of normal attraction. The question if the analogue of this is also true for positively graduated simply connected nilpotent Lie groups (a natural framework for the generalization of the concept of stability to the non-commutative case) turns out to be non-trivial. The reason is that, in this case, non-strict stability is defined in terms of generating distributions of continuous one-parameter convolution semigroups rather than just for the laws themselves. We show that the answer is affirmative for non-degenerate (not necessarily strictly) α-dilation-stable laws on simply connected step 2-nilpotent Lie groups (so, e.g., all Heisenberg groups and all so-called groups of type H; cf. Kaplan [6]) if $\alpha \in]0, 1[\cup]1, 2]$. The proof generalizes to positively graduated simply connected Lie groups which are nilpotent of higher step if $\alpha \in]0, 1[$.