EMPIRICAL PROCESSES, VAPNIK-CHERVONENKIS CLASSES AND POISSON PROCESSES

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Abstract: For background of this paper see [2]. Given a probability space \((X, \mathcal{A}, P)\), let \(G_P\) be the Gaussian process with mean 0, indexed by \(\mathcal{A}\), and such that

\[
EG_P(A)G_P(B) = P(A \cap B) - P(A)P(B), \quad A, B \in \mathcal{A}.
\]

(1) Let \(C \subset \mathcal{A}\) and suppose that, for all probability measures, (laws) \(Q\) on \(\mathcal{A}\), \(G_Q\) has a version with bounded sample functions on \(C\). (For example, suppose \(C\) is a "universal Donsker class".) Then, for some: \(n\), no set \(F\) of \(n\) elements has all its subsets of the form \(C \cap F, C \in C\), i.e. \(C\) is a Vapnik-Chervonenkis class. An example shows that limit theorems for empirical measures need not hold uniformly over a Vapnik-Chervonenkis class of measurable sets, unless further measurability is assumed.

(2) For a law \(P\) on \(X = \{1, 2, \ldots\}\), the collection \(2^X\) of all subsets is a Donsker class if and only if

\[
\sum_{m=1}^{\infty} P(m)^{1/2} < \infty.
\]

(3) For any probability space \((X, \mathcal{A}, P)\), suppose \(C\) is a P-Donsker class, \(C \in \mathcal{A}\). Let \(T_\alpha\) be a Poisson point process with intensity measure \(\alpha P\), \(\alpha > 0\). Then, as \(\alpha \to \infty\), \((T_\alpha - aP)/a^{1/2}\) converges in law, with respect to uniform convergence on \(C\), to the Gaussian process \(W_P\) with mean 0 and \(EW_P(A)W_P(B) = P(A \cap B), \quad A, B \in C\).

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