ON CONVERGENCE OF $L_1$-BOUNDED MARTINGALES INDEXED BY DIRECTED SETS

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Abstract: Let $(F_t)$ be an increasing family of $\sigma$-algebras indexed by a directed set $J$. In this paper it is shown that every $L_1$-bounded real-valued martingale converges essentially if and only if a weak type of maximal inequality holds for all martingales. A new covering condition $C$ stated in terms of multivalued stopping times is introduced and characterized in terms of maximal inequalities. $C$ is shown to be strictly weaker than the Vitali condition $V$, than $SV$ (see [15]), and also sigma-$SV$. Under $C$, $L_1$-bounded martingales taking values in a Banach space with the Radon-Nikodým property converge essentially.

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