REMARKS ON THE POSNVTYTY OF DENSITIES OF STABLE LAWS

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Abstract: Let $0 < \alpha < \infty$, $\alpha \neq 1$, and $S$ be a non-empty subset of $\mathbb{R}^d$, the $d$-dimensional Euclidean space. It is shown that if $S$ satisfies $aS + bS = S$ whenever $a, b \geq 0$ with $a^{\alpha} + b^{\alpha} = 1$, then $S$ is a convex cone with vertex at 0. This, in particular, confirms a conjecture of Port and Vitale [4]. Using this result, an elementary, completely geometric and unified proof is provided for the following known result concerning, the positivity properties of densities of $\alpha$-stable laws on $\mathbb{R}^d$, $0 < \alpha < 2$, $\alpha \neq 1$: Let $X$ be a strictly $\alpha$-stable random vector in $\mathbb{R}^d$ with truly $d$-dimensional law $\mu$, and let $p(t, \cdot)$ and $\sigma$ be the density of $t^{1/\alpha}\mu$, the law $t^{1/\alpha}X$, and the spectral measure of $\mu$, respectively. If $0 < \alpha < 1$ and the support of $\sigma$ is contained in a half-space, then, for any $t > 0$, $p(t, x) > 0$ if and only if $x$ belongs to the interior of the convex cone generated by support of $\sigma$; and, in all other cases, $p(t, x) > 0$ for all $t > 0$ and $x \in \mathbb{R}^d$.

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