ON THE KOLMOGOROV QUASIMARTINGALE PROPERTY

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Abstract: Let \((X_k)\) be a sequence of real-valued random variables (r.v.), which are centered, square integrable and independent. A well-known result, due to Kolmogorov, states that if

\[
\sum_{k \geq 1} \frac{E(X_k^2)}{k^2} < +\infty, \tag{i}
\]

then \((S_n/n)\) converges almost surely (a.s.) to 0, where \(S_n = X_1 + \ldots + X_n\).

This paper is devoted to the interpretation of condition (i). For instance, it is shown that if the r.v. \(X_k\) are weighted Rademacher r.v., then (i) is equivalent to the fact that \(((S_n/n)^2, G_n)\) is a quasimartingale (\(G_n\) being the natural filtration associated with the sequence \((X_n)\)).

The problem of the interpretation of (i) for Banach space valued r.v. \(X_k\) is also studied.

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