LEVEL CROSSINGS AND LOCAL TIME FOR REGULARIZED GAUSSIAN PROCESSES

Corinne Berzin  
José R. León  
Joaquín Ortega

Abstract: Let \( \{X_n, t \in [0, 1]\} \) be a centred stationary Gaussian process defined on \((\Omega, A, P)\) with covariance function satisfying
\[
r(t) \sim 1 - C|t|^{2\alpha}, \quad 0 < \alpha < 1, \quad \text{as } t \to 0.
\]
Define the regularized process
\[
X^\varepsilon = \varphi_\varepsilon \ast X \quad \text{and} \quad Y^\varepsilon = X^\varepsilon / \sigma_\varepsilon, \quad \text{where } \sigma_\varepsilon^2 = \text{var}X^\varepsilon,
\]
\(\varphi_\varepsilon\) is a kernel which approaches the Dirac delta function as \(\varepsilon \to 0\) and \(\ast\) denotes the convolution. We study the convergence of
\[
Z_\varepsilon(f) = \varepsilon^{-a(\alpha)} \int_{-\infty}^{\infty} \left[ \frac{N^V(x)}{c(\varepsilon)} - L_x(x) \right] f(x)dx \quad \text{as } \varepsilon \to 0,
\]
where \(N^V(x)\) and \(L_V(x)\) denote, respectively, the number of crossings and the local time at level \(x\) for the process \(V\) in \([0, 1]\) and
\[
c(\varepsilon) = \left( \frac{2\text{var}(X^\varepsilon)}{\pi \text{var}(X^\varepsilon)} \right)^{1/2}.
\]
The limit depends on the value of \(\alpha\).

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