OPERATORS ON MARTINGALES, Φ-SUMMING OPERATORS, AND THE
CONTRACTION PRINCIPLE

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Abstract: For the absolutely Φ-summing operators $T : X \to Y$ between Banach
spaces $X$ and $Y$ we consider martingale inequalities of the type

$$\left\| \sup_{1 \leq k \leq N} \left\| \sum_{l=1}^{k} T d_l \right\|_{L_2^Y} \right\|_{L_2^X} \leq \frac{c}{\sqrt{r}} \left\| \sum_{i=1}^{n} |\langle d_k, a_i \rangle|^2 \right\|_{L_2^X}$$

where $(d_k)_{k=0}^{N} \subset L_1^X(\Omega, \mathcal{F}, P)$ is a martingale difference sequence and $(a_i)_{i=1}^{\infty}$ is
a sequence of normalized functionals on $X$, and we show that these inequalities are
useful in different directions. For example, for a Banach space $X$, $x_1, \ldots, x_n \in X,$
independent standard Gaussian variables $g_1, \ldots, g_n,$ and $1 \leq r < \infty$ we deduce that

$$\left\| \sum_{i=1}^{n} \left[ \sum_{k=\tau_{i-1}+1}^{\tau_i} d_k \right] x_i \right\|_{L_r^X} \leq c \sqrt{r} \left\| \sum_{1 \leq i \leq n} S_2(\tau_{i-1} f_{\tau_i}) \right\|_{L_r^Y} \left\| \sum_{i=1}^{n} g_i x_i \right\|_{L_r^X},$$

where $f = (d_k)_{k=0}^{N}$ is a scalar-valued martingale difference sequence such that
$(|d_k|)_{k=1}^{N}$ is predictable, $0 = \tau_0 \leq \tau_1 \leq \ldots \leq \tau_n = N$ is a sequence of stopping
times, and

$$S_2(\tau_{i-1} f_{\tau_i}) := \left( \sum_{k=\tau_{i-1}+1}^{\tau_i} |d_k|^2 \right)^{1/2}.$$

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