INDEPENDENT MARGINALS OF OPERATOR-SEMISTABLE AND OPERATOR-STABLE PROBABILITY MEASURES*

BY

ANDRZEJ ŁUCZAK (Łódź)

Abstract. We investigate independent marginals of full operator-semistable and operator-stable probability measures on finite-dimensional vector spaces. In particular, it is shown that for purely Poissonian operator-semistable and operator-stable distributions their independent marginals have decomposability properties of the same kind. Operator-semistability and operator-stability of independent marginals of Gaussian measures are studied in detail, and a description of independent marginals of an arbitrary operator-semistable or operator-stable distribution is obtained.

Introduction. Let \( \mu \) be a probability measure on a finite-dimensional real vector space \( V \) with \( \sigma \)-algebra \( \mathcal{B}(V) \) of its Borel subsets. A projection \( T \) on \( V \) will be called an independent marginal of \( \mu \) if

\[
\mu = T\mu*(I-T)\mu \quad (I \text{ – the identity operator}),
\]

i.e. if \( T \) and \( I-T \) are independent random variables from probability space \( (V, \mathcal{B}(V), \mu) \) into \( V \) (the same name will be sometimes applied also to the measure \( T\mu \)). The aim of the paper is to investigate properties of measure \( T\mu \) for \( T \) being an independent marginal of \( \mu \), and \( \mu \) being a full operator-semistable or operator-stable probability distribution on \( V \). Problems of this type have been considered in [2], [6], and [9], and in this work we generalize and complete some of the earlier results. In particular, we show that for purely Poissonian operator-semistable and operator-stable distributions their independent marginals follow, in principle, the same pattern of decomposability. Operator-semistability and operator-stability of independent marginals of Gaussian measures are studied in detail, and, finally, a description of independent marginals of an arbitrary operator-semistable or operator-stable distribution is obtained.

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1. Preliminaries and notation. Throughout the paper, \( V \) will stand for an \( r \)-dimensional real vector space with an inner product \((\cdot, \cdot)\) yielding a norm \( \| \cdot \| \), and the algebra \( \mathcal{B}(V) \) of its Borel subsets.

An infinitely divisible measure \( \mu \) on \( V \) has the unique representation \([m, D, M]\), where \( m \in V, D \) is a non-negative linear operator on \( V \), and \( M \) is the Lévy spectral measure of \( \mu \), i.e. a Borel measure defined on \( V - \{0\} \) such that

\[
\int_{V - \{0\}} \|u\|^2/(1 + \|u\|^2) \, M(du) < \infty.
\]

The characteristic function \( \hat{\mu} \) of \( \mu \) takes then the form

\[
\hat{\mu}(v) = \exp \left\{ i(m, v) - \frac{1}{2} (Dv, v) + \int_{V - \{0\}} \left[ e^{i(v, u)} - 1 - \frac{i(v, u)}{1 + \|u\|^2} \right] \, M(du) \right\}
\]
(see e.g. [7]). The measure \([m, D, 0]\) is called the Gaussian part of \( \mu \), the measure \([0, 0, M]\) is called its Poissonian part; \( \mu \) is called purely Gaussian if \( M = 0 \), and purely Poissonian if \( D = 0 \).

A probability measure on \( V \) is called full if it is not concentrated on any proper hyperplane of \( V \).

The main objects of our considerations will be full operator-semistable and operator-stable probability measures on \( V \) and their independent marginals as defined in the Introduction. For a more detailed description of these measures, the reader is referred to [3] and [5] (operator-semistable) and [1], [4] and [8] (operator-stable). Here we only recall that if \( \mu \) is a full operator-semistable measure, then it is infinitely divisible and

\[
\mu^a = A \mu \ast \delta(h)
\]
for some \( 0 < a < 1, \ h \in V \), and a non-singular linear operator \( A \) in \( V \). Measures satisfying (1) will be called \((a, A)\)-quasi-decomposable, and for full measures quasi-decomposability is equivalent to operator-semistability. Furthermore, there are decompositions

\[
\mu = \mu_1 \ast \mu_2, \quad V = V_1 \oplus V_2
\]

such that \( V_1 \) and \( V_2 \) are \( A \)-invariant subspaces of \( V \), \( \mu_1 \) is a purely Poissonian \((a, A)\)-quasi-decomposable measure concentrated (and full) on \( V_1 \), and \( \mu_2 \) is a Gaussian \((a, A)\)-quasi-decomposable measure concentrated (and full) on \( V_2 \).

We let \( G_a(\mu) \) denote the set of the operators \( A \)'s which can occur in equation (1).

Full operator-stable measures are characterized by the following condition:

There exists a non-singular operator \( B \) in \( V \), called an exponent of \( \mu \), such that for each \( t > 0 \)

\[
t^B \in G_a(\mu), \quad \text{where} \ t^B = e^{(\log t)B}.
\]
Moreover, decompositions (2) also hold with $\mu_1$ and $\mu_2$ being $B$-invariant, $\mu_1$ — purely Poissonian concentrated on $V_1$, $\mu_2$ — Gaussian concentrated (and full) on $V_2$, and for $i = 1, 2$

$$\mu_i^t = t^B \mu_i \circ \delta(h^{(i)}_t), \quad t > 0,$$

with some $h^{(i)}_t \in V_i$.

2. Marginals of operator-semistable measures. We begin with the following generalization of Theorem 6 of [6].

**Theorem 1.** Let $\mu = [m, 0, \mathcal{M}]$ be a full $(a, A)$-quasi-decomposable probability measure on $V$, and let $T$ be an independent marginal of $\mu$. Then there exists a positive integer $n$ such that $TA^n = A^n T$, and, consequently, $T\mu$ is $(a^n, A^n)$-quasi-decomposable.

**Proof.** Put

$$U = T(V), \quad W = (I - T')(V),$$

and let $S_M$ be the support of the Lévy measure $M$. By virtue of [6] and [9] we have

$$S_M \subset U \cap W.$$

From the fullness of $\mu$, and thus $M$, it follows that $\text{Lin} S_M = V$ and, consequently,

$$\text{Lin}(S_M \cap U) = U, \quad \text{Lin}(S_M \cap W) = W.$$

Equality (1) implies that $aM = AM$, which in turn yields the $A$-invariance of $S_M$.

Let $\{v_1, \ldots, v_k\} \subset S_M \cap U$ be a basis in $U$, and let $\{v_{k+1}, \ldots, v_r\} \subset S_M \cap W$ be a basis in $W$ (we have assumed that $\dim U = k$ and $\dim W = r - k$). According to (3) and the $A$-invariance of $S_M$, for each $m = 0, 1, \ldots$ and each $i = 1, \ldots, r$, $A^m v_i$ is either in $S_M \cap U$ or in $S_M \cap W$. Let us represent the sequence

$${A^m v_1, \ldots, A^m v_k, A^m v_{k+1}, \ldots, A^m v_r}$$

as a sequence of 0's and 1's, where 0 at the $i$-th place means that $A^m v_i \in S_M \cap U$ and 1 at the $i$-th place means that $A^m v_i \in S_M \cap W$ (for instance, if $m = 0$, we have the sequence $\{0, \ldots, 0, 1, \ldots, 1\}$). Condition (3) together with the fullness of $M$ implies that exactly $k$ elements of

$${A^m v_1, \ldots, A^m v_j}$$

are in $S_M \cap U$, and $r - k$ elements are in $S_M \cap W$; in other words, in our representing sequences there will be exactly $k$ zeros and $r - k$ ones. Since there are only \(r \choose k\) such different sequences, we can find elements $v_{i_1}, \ldots, v_{i_k}$ and two positive integers $m_1, m_2$,

$$m_1 < m_2, \quad m_2 - m_1 \leq \binom{r}{k},$$

such that

$$A^{m_1} v_{i_1}, \ldots, A^{m_1} v_{i_k} \in U \quad \text{(the zeros)},$$

$$A^{m_1} v_j \in W \text{ for } j \notin \{i_1, \ldots, i_k\} \quad \text{(the ones)}$$

and

$$A^{m_2} v_{i_1}, \ldots, A^{m_2} v_{i_k} \in U, \quad A^{m_2} v_j \in W \text{ for } j \notin \{i_1, \ldots, i_k\}.$$
Putting
\[ u_1 = A^{m_1} v_{i_1}, \ldots, u_k = A^{m_1} v_{i_k}, \quad w_j = A^{m_1} v_j \text{ for } j \notin \{i_1, \ldots, i_k\} \]
and \( n = m_2 - m_1 \), we get
\[ u_1, \ldots, u_k \in U, \quad A^n u_1, \ldots, A^n u_k \in U \]
and
\[ w_j \in W, \quad A^n w_j \in W \text{ for } j \notin \{i_1, \ldots, i_k\}. \]
Since \( \{u_1, \ldots, u_k\} \) form a basis in \( U \) and \( \{w_j\} \) form a basis in \( W \), we obtain
\[ A^n(U) = U, \quad A^n(W) = W, \]
showing that \( TA^n = A^n T \).

Iterating equality (1) gives the formula
\[ \mu^n = A^n \mu \delta(h_n), \]
and, consequently,
\[ (T^e)^n = T\mu^n = TA^n \mu \delta(T h_n) = A^n T \mu = A^n T h^n \delta(T h_n), \]
so \( T \mu \) is \((a^n, A^n)\)-quasi-decomposable.

Our next aim is to investigate \((a, A)\)-quasi-decomposable Gaussian measures. We begin with a simple characterization of operators \( A \)'s for which a full Gaussian distribution can be \((a, A)\)-quasi-decomposable.

**Proposition 2.** Let \( \mu = [m, D, 0] \) be a full Gaussian measure on \( V \), and let \( a > 0 \). Then
\[ G_a(\mu) = \sqrt{aD^{-1/2}} OD^{-1/2}, \]
where \( O \) is the orthogonal group on \( V \).

**Proof.** It is easy to verify that a Gaussian measure \( \mu = [m, D, 0] \) satisfies equation (1) if and only if
\[ aD = ADA^*. \]
It is immediately seen that for any orthogonal \( H \) and the operator \( A \) defined as
\[ A = \sqrt{aD^{1/2}} HD^{-1/2} \]
equality (4) holds, which proves the inclusion
\[ \sqrt{aD^{1/2}} OD^{-1/2} \subset G_a(\mu). \]

Assume now that (4) holds. The fullness of \( \mu \) implies the invertibility of \( D \), and we have
\[ aI = D^{-1/2} ADA^* D^{-1/2} = (D^{1/2} A^* D^{-1/2})^* D^{1/2} A^* D^{-1/2}, \]
which means that the absolute value of the operator \( D^{1/2} A^* D^{-1/2} \) is \( \sqrt{aI} \). The polar decomposition formula gives the equality
\[ D^{1/2} A^* D^{-1/2} = H |D^{1/2} A^* D^{-1/2}| = \sqrt{aH} \]
for some orthogonal $H$, so

$$A = (\sqrt{aD^{-1/2}} HD^{-1/2})^* = \sqrt{aD^{1/2}} H^* D^{-1/2},$$

showing that $A \in \sqrt{aD^{1/2}} OD^{-1/2}$. 

\textbf{Remark.} The above proposition can be thought of as an “operator-semi-stable” counterpart of Theorem 4.6.10 from [4], which gives a characterization of the set of exponents of Gaussian measures.

Now we shall analyse conditions of quasi-decomposability of independent marginals of full Gaussian measures.

**Proposition 3.** Let $\mu = [m, D, 0]$ be a full $(a, A)$-quasi-decomposable Gaussian measure on $V$, and let $T$ be an independent marginal of $\mu$. Then $T\mu$ is $(a, A)$-quasi-decomposable if and only if $A$ and $T$ commute.

**Proof.** Put $P = I - T$. Then

$$\mu^a = (T\mu \ast P\mu)^a = (T\mu)^a \ast (P\mu)^a = T\mu^a \ast P\mu^a$$

and

$$A\mu = AT\mu \ast AP\mu.$$ 

From equality (1) we get

$$T\mu^a \ast P\mu^a = AT\mu \ast AP\mu \ast \delta(h);$$

thus

$$T\mu^a = TAT\mu \ast TAP\mu \ast \delta(Th).$$

(5)

If $A$ and $T$ commute, we have $TAP = 0$, so (5) becomes

$$T\mu^a = AT\mu \ast \delta(Th),$$

which means that $T\mu$ is $(a, A)$-quasi-decomposable.

Now, assume that $T\mu$ is $(a, A)$-quasi-decomposable. Then

$$T\mu^a = AT\mu \ast \delta(h'),$$

so

$$T\mu^a = TAT\mu \ast \delta(Th'),$$

which together with (5) leads to the equality

$$TAT\mu \ast \delta(Th') = TAT\mu \ast TAP\mu \ast \delta(Th).$$

Since all the measures involved are Gaussian, the above equality shows that $TAP\mu$ is a degenerate measure and, consequently,

$$\text{Note.}$$

(6)

$$(TAP)D(TAP)^* = 0.$$ 

By Proposition 2, $A$ takes the form $A = \sqrt{aD^{1/2}} HD^{-1/2}$ for some orthogonal $H$, so (6) leads to the equality

$$aTD^{1/2} HD^{-1/2} PDP^* D^{-1/2} H^* D^{1/2} T^* = 0,$$
and multiplying on the left by $D^{-1/2}$ and on the right by $D^{1/2}$, we get

\[ D^{-1/2} T D^{1/2} H D^{-1/2} P D^{1/2} P^* D^{-1/2} H^* T D^{1/2} = 0. \]

Put

\[ D^{-1/2} T D^{1/2} = R. \]

Then $R = R^2$; moreover,

\[ R^* = D^{1/2} T^* D^{-1/2} = D^{-1/2} D T^* D^{-1/2}. \]

Since $T$ is an independent marginal, we have, according to [6] and [9],

\[ D = T D T^* + P D P^*, \]

so

\[ T D = T D T^* = D T^*. \]

Thus (8) leads to the equality

\[ R^* = D^{-1/2} T D D^{-1/2} = D^{-1/2} T D^{1/2} = R, \]

showing that $R$ is an orthogonal projection. Furthermore,

\[ R^\perp = I - R = D^{-1/2} (I - T) D^{1/2} = D^{-1/2} P D^{1/2}. \]

Consequently, equality (7) takes the form $R H R^\perp R = 0$, so

\[ R H R^\perp (R H R^\perp)^* = 0, \]

which means that

\[ R H R^\perp = 0, \quad \text{i.e.} \quad R H = R H R. \]

Since $H$ is orthogonal and $R$ is an orthogonal projection, the last equality means that $H$ and $R$ commute. Thus we have

\[ D^{-1/2} T D^{1/2} H = H D^{-1/2} T D^{1/2}, \]

which, in turn, gives

\[ T D^{1/2} H D^{-1/2} = D^{1/2} H D^{-1/2} T. \]

Multiplying both sides by $\sqrt{a}$, we finally obtain $T A = A T$, which completes the proof. \( \Box \)

The last two results lead us to an example of a full $(a, A)$-quasi-decomposable Gaussian measure having $r$ independent one-dimensional marginals which are not $(a^n, A^n)$-quasi-decomposable for any $n$.

**Example.** Let $T_1, \ldots, T_r$ be one-dimensional orthogonal projections, and let $0 < \lambda_1 < \ldots < \lambda_r$. Put

\[ D = \sum_{i=1}^r \lambda_i T_i, \]
and let $\mu = [0, D, 0]$. We have

$$D = \sum_{i=1}^{r} T_i D T_i = \sum_{i=1}^{r} T_i D^2 T_i^*;$$

thus $T_1, \ldots, T_r$ are independent marginals of $\mu$. Let $H$ be an orthogonal operator, and put

$$A = \sqrt{aD^{1/2} HD^{-1/2}}$$

for some $a > 0$.

By Proposition 2, $\mu$ is $(a, A)$-quasi-decomposable. Now, for any integer $n$,

$$A^n = a^{n/2} D^{1/2} H^n D^{-1/2},$$

so $A^n$ commutes with $T_i$ if and only if $H^n$ does. Hence, if we have chosen $H$ in such a way that

$$H^n T_i \neq T_i H^n, \quad i = 1, \ldots, r, \text{ all } n,$$

then by Proposition 3 none of the marginals $T_i$'s will be $(a^n, A^n)$-quasi-decomposable for any $n$. 

Our final goal in this chapter is to give a description of independent marginals of an arbitrary full $(a^n, A^n)$-quasi-decomposable measure. We have

**Theorem 4.** Let $\mu = [m, D, M]$ be a full $(a, A)$-quasi-decomposable measure on $V$, and let $T$ be an independent marginal of $\mu$ with $T(V) = U$. Then there are decompositions

$$U = U_1 \oplus U_2, \quad T\mu = v_1 * v_2$$

such that $v_1$ is a purely Poissonian $(a^n, A^n)$-quasi-decomposable (for some $n$) measure concentrated on $U_1$, and $v_2$ is a Gaussian measure concentrated on $U_2$.

**Proof.** Put $P = I - T$, $W = P(V)$, and let again $S_M$ stand for the support of $M$. For $S_M$ relation (3) holds; thus putting

$$U_1 = \text{Lin}(S_M \cap U), \quad W_1 = \text{Lin}(S_M \cap W),$$

we get

$$\text{Lin} S_M = U_1 \oplus W_1.$$ 

Now, let us take into account decompositions (2). The Poissonian part $\mu_1$ lives on $V_1$, so we have $V_1 = U_1 \oplus W_1$. Restrict for the moment our attention to the subspace $V_1$ and the measure $\mu_1$. We have $S_M \subset U_1 \cup W_1$. Thus denoting by $T_1$ the projection onto $U_1$ with kernel $W_1$, and by $P_1$ the projection onto $W_1$ with kernel $U_1$, we infer from [6] and [9] that $T_1$ and $P_1$ are independent marginals of $\mu_1$, so by Theorem 1 we have

$$T_1 A^n = A^n T_1, \quad P_1 A^n = A^n P_1$$

for some $n$,

and $T_1 \mu_1, P_1 \mu_1$ are $(a^n, A^n)$-quasi-decomposable.
Now we shall analyse the Gaussian part. It is concentrated on $V_2$, so we have

$$D(V) = D(V_2) = V_2.$$ 

Since $T$ and $P$ are independent marginals of $\mu$, relation (9) holds. Thus

$$T(V_2) = TD(V_2) = DT^*(V_2) \subset V_2$$

and, similarly,

$$P(V_2) \subset V_2.$$ 

Putting $T(V_2) = U_2$ and $P(V_2) = W_2$, we obtain the decomposition $V_2 = U_2 \oplus W_2$. Let $R$ be the orthogonal projection onto $V_2$. We have $D = RD$, so $R$ and $D$ commute. Furthermore,

$$(T \mid V_2)^* = RT^* \mid V_2, \quad (P \mid V_2)^* = RP^* \mid V_2,$$

which together with the equality

$$D = TDRT^* + PDP^*$$

gives

$$D \mid V_2 = TDRT^* \mid V_2 + PDP^* \mid V_2$$

$$= (T \mid V_2)(D \mid V_2)(T \mid V_2)^* + (P \mid V_2)(D \mid V_2)(P \mid V_2)^*.$$

Now restricting our attention to the subspace $V_2$ and the measure $\mu_2$, and denoting by $T_2$ the projection onto $U_2$ with kernel $W_2$, and by $P_2$ the projection onto $W_2$ with kernel $U_2$, we get

$$D = T_2DT_2^* + P_2DP_2^*,$$

which means that $T_2$ and $P_2$ are independent marginals of $\mu_2$. Finally, we have

$$V = V_1 \oplus V_2 = (U_1 \oplus W_1) \oplus (U_2 \oplus W_2) = (U_1 \oplus U_2) \oplus (W_1 \oplus W_2) = U \oplus W,$$

and since

$$U_1 \oplus U_2 \subset U, \quad W_1 \oplus W_2 \subset W,$$

we obtain

$$U = U_1 \oplus U_2, \quad W = W_1 \oplus W_2.$$ 

Extending the projections $T_1$, $T_2$, $P_1$, $P_2$ in the natural way to the whole $V$ (i.e. for instance $T_1$ will be the projection onto $U_1$ with kernel $U_2 \oplus W_1 \oplus W_2$) we shall get

$$T = T_1 + T_2, \quad P = P_1 + P_2$$

and

$$\mu = \mu_1 \ast \mu_2 = T_1 \mu_1 \ast P_1 \mu_1 \ast T_2 \mu_2 \ast P_2 \mu_2,$$

which gives

$$T_i \mu = T_i \mu_i, \quad P_i \mu = P_i \mu_i, \quad i = 1, 2.$$ 

Thus we have

$$\mu = T \mu \ast P \mu = T_1 \mu \ast P_1 \mu \ast T_2 \mu \ast P_2 \mu,$$
and applying $T$ to both sides of the above equality we obtain $T\mu = T_1\mu * T_2\mu$. Putting $v_1 = T_1\mu$ and $v_2 = T_2\mu$, we obtain the desired decomposition. ■

Remark. Neither the measure $v_2$ nor the measure $P_2\mu$ need not be $(a^n, A^n)$-quasi-decomposable for any $m$ (however, their convolution being the Gaussian part $\mu_2$ of $\mu$ is $(a, A)$-quasi-decomposable). Nevertheless, this fact does not affect operator-semistability of the marginal $T\mu$ as is seen in the following corollary.

COROLLARY. Let $T$ be an independent marginal of a full $(a, A)$-quasi-decomposable measure $\mu$ on $V$. Then $T\mu$ is operator-semistable.

Proof. In the course of the proof of Theorem 4 it was shown that $T\mu = T_1\mu * T_2\mu$ with $T_1 A^n = A^n T_1$ for some $n$, which means that $A^n(U_1) = U_1$. Define an operator $A_n$ by

$$A_n = \begin{cases} A^n & \text{on } U_1, \\ \sqrt{a^n} I & \text{on } U_2, \\ \text{arbitrary} & \text{on } W. \end{cases}$$

Since $T_2\mu$ is Gaussian, it is $(a^n, \sqrt{a^n} I)$-quasi-decomposable, and we have

$$(T\mu)^{a^n} = (T_1\mu)^{a^n} * (T_2\mu)^{a^n} = A^n T_1\mu * \delta(h_1) * \sqrt{a^n} T_2\mu * \delta(h_2)$$

$$= A_n T_1\mu * A_n T_2\mu * \delta(h_1 + h_2) = A_n (T_1\mu * T_2\mu) * \delta(h_1 + h_2)$$

$$= A_n T\mu * \delta(h_1 + h_2),$$

showing that $T\mu$ is $(a^n, A_n)$-quasi-decomposable, hence operator-semistable. ■

3. Marginals of operator-stable measures. In general, operator-stability exhibits much more regular behaviour as will be seen in the following counterparts of results about operator-semistability. In particular, we have

THEOREM 5. Let $\mu = [m, 0, M]$ be a full operator-stable probability measure on $V$ with exponent $B$, and let $T$ be an independent marginal of $\mu$. Then $T$ and $B$ commute, and $T\mu$ is operator-stable with exponent $TB$.

Proof. Putting $U = T(V)$ and $W = (I - T)(V)$, we have again relation (3), and the equality $\mu' = t^B \mu * \delta(h)$ yields the inclusion $t^B(S_M) \subseteq S_M$. Thus, for an arbitrary $u \in S_M \cap U$, $t^B u \in S_M \cap W$, and the same is true for $w \in W$. From the fullness of $M$ we infer that $t^B(U) \subset U$ and $t^B(W) \subset W$, and differentiation at 1 gives $B(U) \subset U$ and $B(W) \subset W$. Since $B$ is invertible, we get $B(U) = U$ and $B(W) = W$, which means that $T$ and $B$ commute. Accordingly,

$$(T\mu)' = T t^B \mu * \delta(T h) = t^{TB} T \mu * \delta(T h),$$

showing that $TB$ is an exponent of $T\mu$. ■
PROPOSITION 6. Let $\mu = [m, D, 0]$ be a full operator-stable Gaussian measure on $V$ with exponent $B$, and let $T$ be an independent marginal of $\mu$. Then $T\mu$ is operator-stable with exponent $TBT$.

Proof. According to Propositions 4.3.2 and 4.3.3 of [4], $B$ is an exponent of $\mu$ if and only if

$$D = BD + DB^*.$$ 

Multiplying the above equality by $T$ on the left and by $T^*$ on the right and taking into account the relations $TD = DT^* = TDT^*$ which follow from (9), we obtain

$$TDT^* = TBDT^* + TDB^*T^* = (TBT)(TDT^*) + (TDT^*)(TBT)^*.$$ 

Since $TDT^*$ is the covariance operator of the measure $T\mu$, applying again the above-mentioned propositions from [4], we see that $TBT$ is an exponent of $T\mu$. □

By reasoning in a similar fashion to that in the proof of Theorem 4, we obtain the following result:

THEOREM 7. Let $\mu$ be a full operator-stable measure on $V$ with exponent $B$, and let $T$ be an independent marginal of $\mu$ with $T(V) = U$. Then there are decompositions

$$U = U_1 \oplus U_2, \quad T\mu = v_1 * v_2$$

such that $v_1$ is a purely Poissonian operator-stable measure concentrated on $U_1$ with exponent $T_1B = BT_1$, and $v_2$ is an operator-stable Gaussian measure concentrated on $U_2$ with exponent $T_2B^2T_2$, where $T_1$ and $T_2$ are projections onto $U_1$ and $U_2$, respectively, with kernels $\ker T_1 = U_2 \oplus W$, $\ker T_2 = U_1 \oplus W$, $W = (I - T)(V)$. □

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Faculty of Mathematics, Łódź University
ul. Stefana Banacha 22, 90-238 Łódź, Poland
E-mail: anluczak@math.uni.lodz.pl

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