Abstract. For a real $p$ ($1 < p < 2$) and its conjugate $p'$ we characterize Banach spaces $E$ for which an operator $T : L_{p'} \to E$ is $\theta_p$-Radonifying iff $T^*$ is $p$-absolutely summing. In case $p = 2$ these are exactly spaces of type 2 as was proved by Chobanjan and Tarieladze [1]. For $p < 2$ the condition is much stronger because these are spaces of stable type $p$ isomorphic to a subspace of some $L_p$.

Let $E$ be a real Banach space. For a real number $p$ ($1 < p < 2$) let $L_p$ be a separable Banach space of measurable functions having $p$-integrable absolute value. Let $1/p + 1/p' = 1$. An operator $T$ from $L_{p'}$ into $E$ is said to be $\theta_p$-Radonifying if $\exp(-\|T'a\|^p)$ is the characteristic function of a Radon measure $\mu$ on $E$. Here $\theta_p$ is a cylindrical measure on $L_{p'}$ with the characteristic function of the form $\exp(-\|g\|^p)$, $g \in L_p$. Thus $T$ is $\theta_p$-Radonifying iff $T(\theta_p)$ extends to a Radon measure on $E$. In this case the Radon extension is a $p$-stable symmetric measure on $E$. It turns out that the set $\Sigma_p(L_{p'}, E)$ of all $\theta_p$-Radonifying operators becomes a Banach space under the equivalent norms

$$\sigma_p(T) = \left( \int_E \|x\|^r \, d\mu \right)^{1/r}, \quad 1 \leq r < p < 2.$$
for each \( r \) with \( 0 < r < p \), where \( \xi_1, \xi_2, \ldots, \xi_n \) is a sequence of i.i.d.
random variables with characteristic function \( \exp(-|t|^p) \).

If \( E \) and \( F \) are Banach spaces, then the operator \( T : E \to F \) is called
\( p \)-\textit{absolutely summing} \((T \in \Pi_p(E,F))\) if for some constant \( M \) and for each
\( x_1, x_2, \ldots, x_n \in E \) the inequality

\[
\sum_{i=1}^{n} \|Tx_i\|^p \leq M^p \sup_{x' \in E', \|x\| \leq 1} \sum_{i=1}^{n} |\langle x_i, x' \rangle|^p
\]

holds. Denote by \( \pi_p(T) \) the least such constant \( M \).

The following relation between the \( \theta_p \)-\textit{Radonifying operators} and the
\( p \)-\textit{absolutely summing operators} is known in a more general version as
the celebrated L. Schwartz's duality theorem (cf. [2] and references therein):

\textbf{Proposition.} If \( T \in \Sigma_p(L_p', E) \), then \( T' \in \Pi_p(E', L_p) \).

The converse implication does not hold in general. In the following
theorem we characterize Banach spaces for which it holds.

\textbf{Theorem.} Let \( 1 < p < 2 \). Then the following two conditions on a Banach
space \( E \) are equivalent:

\begin{enumerate}
  \item \( T \in \Sigma_p(L_p', E) \) if \( T' \in \Pi_p(E', L_p) \) for each space \( L_p' \).
  \item \( E \) is of \textit{stable type} \( p \) and isomorphic to a subspace of some \( L_p \).
\end{enumerate}

\textbf{Proof.} (1) \( \Rightarrow \) (2). Let \( x_1, x_2, \ldots, x_n \in E \). We define an operator \( T \) from
\( L_p' \) into \( E \) by

\[
\| T'a \|^p = \sum_{i=1}^{n} |\langle x_i, a \rangle|^p.
\]

Condition (1) implies the existence of a constant \( c > 0 \) such that
\( \sigma_{p'}(T) \leq c \pi_p(T') \). In addition, let us observe that the characteristic function
of the \( p \)-\textit{stable measure} \( \mu \) defined by \( T \) is equal to the characteristic
function of the distribution of the \( E \)-valued random vector \( \sum_{i=1}^{n} x_i \xi_i \). Namely,

\[
\tilde{\mu}(a) = \exp(-\| T'a \|^p) = \exp(-\sum_{i=1}^{n} |\langle x_i, a \rangle|^p).
\]

Thus we have

\[
(E \| \sum_{i=1}^{n} x_i \xi_i \|^r)^{1/r} = (\int_E \| x \|^r d\mu)^{1/r} = \sigma_{p'}(T) \leq c \pi_p(T') \leq c \big( \sum_{i=1}^{n} \| x_i \|^p \big)^{1/p},
\]

which shows that \( E \) is of \textit{stable type} \( p \).

To prove that the space \( E \) is isomorphic to a subspace of some \( L_p \)
we choose \( x_1, x_2, \ldots, x_n \) and \( y_1, y_2, \ldots, y_n \) belonging to \( E \) with the property

\[
\sum_{i=1}^{n} |\langle x_i, a \rangle|^p \leq \sum_{i=1}^{n} |\langle y_i, a \rangle|^p \quad \text{for all} \quad a \in E'.
\]
Radonifying operators

Now we define operators $T$ and $S$ from $L_p$ into $E$ by

$$\| T'a \|^p = \sum_{i=1}^n |\langle x_i, a \rangle|^p \quad \text{and} \quad \| S'a \|^p = \sum_{i=1}^n |\langle y_i, a \rangle|^p \quad \text{for all } a \in E'.$$

The inequality $\| T'a \| \leq \| S'a \|$ for all $a \in E'$ implies $\pi_p(T') \leq \pi_p(S').$ Since each Banach space is of stable cotype $p$ for $p < 2$, we have

$$\left( \sum_{i=1}^n \| x_i \|_p \right)^{-1/\rho} \leq c_1 \| E \| \left( \sum_{i=1}^n \| x_i \|_p \right)^{-1/\rho} = c_1 \sigma_p(T)$$

$$\leq c_2 \pi_p(T') \leq c_2 \pi_p(S') \leq c_2 \left( \sum_{i=1}^n \| y_i \|_p \right)^{-1/\rho}.$$

By Lindenstrauss-Pelczyński's theorem ([4], Theorem 7.3) we claim that $E$ is isomorphic to a subspace of some $L_p$.

(2) $\Rightarrow$ (1). Consider an operator $T : L_p \rightarrow E$ such that $T'$ is $p$-absolutely summing. Since $E$ is isomorphic to a subspace of some $L_p$, by Kwapień's theorem [3] we have $T \in \Pi_p(L_p, E)$. It follows from separability of the space $L_p$ that there exists an isometric imbedding $J$ from $L_p$ into $L_p[0,1]$. Then $JT'$ is $p$-absolutely summing. By Kwapień's theorem [2] there exists a strongly measurable function $\varphi$ from $[0,1]$ into $E$ with $E\|\varphi\|_p < \infty$ such that

$$\| T'a \|_p = \| JT'a \|_p = \frac{1}{a} \int |\varphi(t), a \rangle|^p dt.$$  

Since $E$ is of stable type $p$, $\exp(-\| T'a \|_p)$ is (by [5]) the characteristic function of a Radon measure, i.e., $T \in \Sigma_p(L_p, E)$, which completes the proof.

**Corollary.** Let $1 < p < 2$. Then the following two conditions on a Banach space $E$ are equivalent:

1. $T \in \Sigma_p(L_p, E)$ if and only if $T' \in \Pi_p(E', L_p)$ for each space $L_p$.
2. $E$ is of stable type $p$ and isomorphic to a subspace of some $L_p$.

**Remark.** It is known by Rosenthal's theorem (see [7]) that condition (2) is equivalent to each of the following ones:

3. $E$ is isomorphic to a subspace of some $L_p$ and does not contain an isomorphic copy of $l_p$.

4. $E$ is isomorphic to a subspace of some $L_p$ and there exists a real $r$ ($0 < r < p$) such that the topologies of $L_p$ and $L_r$ coincide on $E$.

5. There exists a real $q$ ($p < q \leq 2$) such that $E$ is isomorphic to a subspace of some $L_q$.

**Added in proof.** After this note was completed, the authors were made aware of the paper of D. H. Thang and N. D. Tien, *On the extension of stable cylindrical measures*, Acta Math. Vietnam. 5 (1980), p. 169-177, where an equivalent result was established. Methods of proofs are however
different. We also refer the reader to our paper *p*-stable measures and *p*-absolutely summing operators, p. 167-178 in: Lecture Notes in Math. 828 Springer Verlag, 1980, for some additional results on this subject.

REFERENCES


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