THE LADDER VARIABLES OF A MARKOV RANDOM WALK

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Abstract: Given a Harris chain \((M_n)_{n \geq 0}\) on any state space \((S, C)\) with essentially unique stationary measure \(\xi\), let \((X_n)_{n \geq 0}\) be a sequence of real-valued random variables which are conditionally independent, given \((M_n)_{n \geq 0}\), and satisfy
\[
P(X_k \in \cdot | (M_n)_{n \geq 0}) = Q(M_{k-1}, M_k, \cdot)
\]
for some stochastic kernel \(Q : S^2 \times B \to [0, 1]\) and all \(k \geq 1\). Denote by \(S_n\) the \(n\)-th partial sum of this sequence. Then \((M_n, S_n)_{n \geq 0}\) forms a so-called Markov random walk with driving chain \((M_n)_{n \geq 0}\). Its stationary mean drift is given by \(\mu = E\xi X_1\) and assumed to be positive in which case the associated (strictly ascending) ladder epochs
\[
\sigma_0 = \inf\{k \geq 0 : S_k \geq 0\},
\]
\[
\sigma_n = \inf\{k > \sigma_{n-1} : S_k > S_{\sigma_{n-1}}\} \quad \text{for } n \geq 1,
\]
and the ladder heights \(S^*_n = S_{\sigma_n}\) for \(n \geq 0\) are a.s. positive and finite random variables. Put \(M^*_n = M_{\sigma_n}\). The main result of this paper is that \((M^*_n, S^*_n)_{n \geq 0}\) and \((M^*_n, \sigma_n)_{n \geq 0}\) are again Markov random walks (with positive increments, thus so-called Markov renewal processes) with Harris recurrent driving chain \((M^*_n)_{n \geq 0}\). The difficult part is to verify the Harris recurrence of \((M^*_n)_{n \geq 0}\). Denoting by \(\xi^*\) its stationary measure, we also give necessary and sufficient conditions for the finiteness of \(E\xi^* S^*_1\), \(E\xi S^*_1\) and \(E\xi^* \sigma_1\) in terms of \(\mu\) or the recurrence-type of \((M_n)_{n \geq 0}\) or \((M^*_n)_{n \geq 0}\).

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The full text is available here.