EDGEWORTH EXPANSIONS FOR $L$-STATISTICS

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Abstract: We study the approximation by a short Edgeworth expansion of the distribution function of normalized linear combinations

$$\frac{1}{\sqrt{n}} \sum_{j=1}^{n} c_{jn} X_{j:n}$$

of order statistics of $n$ independent random variables with common distribution function $F$. Under the assumptions

$$|c_{jn}| \leq C n^{-p_1} \left[ \frac{j}{n} \left(1 - \frac{j - 1}{n}\right) \right]^{-p_2},$$

$$|c_{jn} - c_{j-1,n}| \leq C n^{-q_1} \left[ \frac{j}{n} \left(1 - \frac{j - 1}{n}\right) \right]^{-q_2},$$

$$|c_{j+1,n} - 2c_{jn} + c_{j-1,n}| \leq C n^{-r_1} \left[ \frac{j}{n} \left(1 - \frac{j - 1}{n}\right) \right]^{-r_2},$$

$$(F^{-1})'(s) \leq C [s(1-s)]^{-\kappa}$$

for some $p_1, q_1, r_1 \in \mathbb{R}$, $p_2, q_2, r_2, C \geq 0$, $\kappa \in [0, 5/4)$, with an appropriate balance in these parameters, and under additional moment conditions, the rate of uniform convergence is shown to be of order $n^{-1}$. Moreover, a special case is considered where the $c_{jn}$ are generated by a sequence of weight functions of a special structure.


Key words and phrases: Linear combinations of order statistics, Edgeworth expansions, rate of convergence.

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