THE RATE OF CONVERGENCE
IN THE PRECISE LARGE DEVIATION THEOREM

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Abstract. Let $X_1, X_2, \ldots$ be i.i.d. random variables with a common d.f. $F$. Let $S_n = X_1 + \ldots + X_n$, $n \geq 1$, and $M_n = \max_{1 \leq k \leq n} X_k$, $n \geq 1$. In this paper for a large class of subexponential distributions we estimate the rate of convergence

$$\Delta_n(t) = P(S_n > t) - P(M_n > t),$$

where $n \geq 1$ and $t \geq 0$. We close this paper with some examples.

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1. INTRODUCTION

Let $X_1, X_2, \ldots$ be i.i.d. real random variables with a common distribution function (d.f.) $F(t)$, $t \in \mathbb{R}$, which has the mean $E X_1 = 0$.

DEFINITION. We say that the d.f. $F$ belongs to the class $S$ of subexponential distributions if its tail $\bar{F} := 1 - F$ satisfies

$$\lim_{t \to \infty} \frac{\bar{F}(x+y)}{\bar{F}(x)} = 1, \quad y \in \mathbb{R},$$

and

$$\lim_{t \to \infty} \frac{\bar{F} \ast \bar{F}(t)}{\bar{F}(t)} = 2,$$

where, as usual, $\ast$ denotes the Stieltjes convolution of $F$ with itself.

The class $S$ of subexponential distributions was introduced by Chistyakov [3] (in the case $F(0) = 0$).

It is well known (see [3], Theorem 2) that if $F(0) = 0$, then (1.2) implies (1.1).
We denote by $\mathfrak{O}$ a class of heavy tailed distributions for which the relation (1.1) is satisfied.

Let $S_n = \sum_{k=1}^{n} X_k$ and $M_n = \max_{k \leq n} X_k$, $n \in \mathbb{N}$.

By definition it follows that if $F \in \mathfrak{S}$, then

$$
P(S_n > t) \sim P(M_n > t) \quad \text{as } t \to \infty.
$$

Thus, we infer that if d.f. $F$ is subexponential, then there exists a positive sequence $t_n$, $n \in \mathbb{N}$, such that

$$
P(S_n > t) \sim P(M_n > t) \quad \text{as } n \to \infty
$$

uniformly in $t \in (t_n, \infty)$.

This means that in the investigation of precise large deviations for subexponential distributions the main problem becomes finding the intervals $(t_n, \infty)$.

Many papers are devoted (see [12] and the references contained therein) to search conditions for which the relation (1.3) holds as $n \to \infty$ uniformly for $t \in (t_n, \infty)$. There are but a few papers that consider the rate of convergence in the relation (1.3). Perhaps the most important paper among them is [2] in which Borovkov has established the rate of convergence in a theorem of large deviations for a class of subexponential distributions, the so-called semiexponential distributions. In the present paper we shall investigate the rate of convergence in (1.3) for one rather wide subclass of subexponential distributions.

2. PRELIMINARIES

Let us define the hazard function $R_F$ of $F$ by

$$
R_F(t) = -\log F(t), \quad t \in \mathbb{R}.
$$

Assume that there exists a non-negative function $q_F: \mathbb{R}^+ \to \mathbb{R}$ such that

$$
R_F(t) = R_F(0) + \int_{0}^{t} q_F(u) \, du, \quad t \in \mathbb{R}^+.
$$

The function $q_F$ is called the hazard rate of $F_0 = F \cdot U_0$, where $U_0$ is the d.f. concentrated at 0.

It is well known (see [7]) that if for some $F_0 \in \mathfrak{O}$ the hazard rate $q_F$ or $\lim_{t \to \infty} q_F(t)$ does not exist, one can always construct a d.f. $H_0$ such that $H_0(t) \sim F_0(t)$ as $t \to \infty$, and $q_H(t) \to 0$ as $t \to \infty$, where $q_H$ is the hazard rate of $H_0$.

Let us define

$$
\alpha = \sup \{ k : \mathbb{E}(X_1^k, X_1 > 0) < \infty \},
$$

$$
\beta = \sup \{ k : \mathbb{E}(|X_1|^k, X_1 < 0) < \infty \}, \quad \gamma = \min(\alpha, \beta).
$$
Moreover, let us define the hazard ratio index

\[ r := \limsup_{t \to \infty} \frac{t q_F(t)}{R_F(t)}. \]

**Lemma 2.1.** Assume that \( \gamma > 2 \) and \( E X_1 = 0 \). Then for \( z > 0 \) we have

\[ \left| \int_{-\infty}^{1/z} e^{zu} dF(u) - 1 \right| < C_0 z^2. \]

**Proof.** We note that

\[ \int_{-\infty}^{1/z} e^{zu} dF(u) - 1 = \int_{-\infty}^{1/z} (e^{zu} - 1 - zu) dF(u) - F(1/z) + z \int_{-\infty}^{1/z} u dF(u). \]

Since \( E X_1 = 0 \), we have

\[ \int_{-\infty}^{1/z} u dF(u) = - \int_{1/z}^{\infty} u dF(u). \]

Hence

\[ \left| \int_{-\infty}^{1/z} e^{zu} dF(u) - 1 \right| \leq z^2 \int_{-\infty}^{1/z} u^2 dF(u) + z \int_{1/z}^{\infty} u dF(u) + F(1/z) \leq 5z^2 E X_1^2. \]

The proof is complete.

### 3. Main Results

In this section we study the rate of convergence in (1.3). For further use, let us define

\[ A_n(t) = P(S_n > t) - P(M_n > t), \]

where \( n \in \mathbb{N} \) and \( t \geq 0 \).

Put \( s := s(t) = R_F(t)/t, \ t > 0. \)

We have

(3.1) \( A_n(t) = P(S_n > t, M_n > t) - P(M_n > t) + P(S_n > t, M_n \leq t) := L_1 + L_2. \)

Our first preliminary result is used to estimate the term \( L_1 \) in (3.1).

**Lemma 3.1.** If \( z > 0 \) is small enough, then

(3.2) \( 0 \geq L_1 \geq -P(M_n > t)(\int t^{-1/z} q_F(u) du + P(|S_n| \geq 1/z) + P(X_n > t)). \)

**Proof.** Let us put \( A^k_n = [1, \ldots, n] \setminus \{k\} \) and \( S_n^k = \sum_{k \in A^k_n} X_k, \ n \in \mathbb{N}. \) From (3.1) it follows that

\[ P(M_n > t) \geq \sum_{k=1}^{n} P(S_n > t, M_n > t, M_n = X_k) \]
\[ \geq \sum_{k=1}^{n} \int_{-\frac{1}{z}}^{\infty} P(X_k > \max(t-u, t)) dP(S_n^k < u) \]

\[ \geq \sum_{k=1}^{n} P(X_k > t + 1/z) P(S_n^k > -1/z) \]

\[ \geq P(M_n > t + 1/z) - P(M_n > t) P(|S_n| > 1/z) - P(M_n > t) P(X_1 > t). \]

Since \( z > 0 \) is small enough, we have

\[ P(M_n > t) - P(M_n > t + 1/z) \]

\[ \leq P(M_n > t) \left(1 - \exp\left(-\int_{t}^{t+1/z} q_F(u) du\right)\right) \leq P(M_n > t) \int_{t}^{t+1/z} q_F(u) du. \]

The proof is complete.

Let \( X_{1:n} \leq X_{2:n} \leq \ldots \leq X_{n-1:n} \leq X_{n:n} = M_n \) denote the order statistics of the sample.

Define

\[ b(r) = \begin{cases} 2 & \text{if } r = 0, \\ 4/(1-r) & \text{if } r \neq 0. \end{cases} \]

Our main result is the following

**THEOREM 3.2.** Assume that

(i) \( EX_1 = 0; \)

(ii) \( \liminf_{t \to \infty} t q_F(t) > 2; \)

(iii) \( r < 1; \)

(iv) \( \beta > 2, \alpha > b(r). \)

Then for \( n \) and \( t \) large enough

\[ -P(M_n > t)(c_0 n^{1-\gamma/2} + c_1 \sqrt{n \log ns}) \leq \Lambda_n(t) \]

\[ \leq P(M_n > t)(\exp(c* n^2) t^2 + C_1 \sqrt{n \log ns + C_2 n^2 + C_3 n^{1-\gamma/2}}), \]

where \( c_0 > 0, c_1 > 0, c* > 0, C_1 > 0, C_2 > 0, C_3 > 0 \) are some constants.

**Remarks.** 1. Let \( t_n, n \in N, \) be a sequence such that

\[ \lim_{n \to \infty} \sqrt{n \log ns(t_n)} = 0. \]

From (3.3) it follows that under the conditions of Theorem 3.2 we have

\[ \Lambda_n(t) = O(1) P(M_n > t) \quad \text{as } n \to \infty \]

uniformly with respect to \( t \in (t_n, \infty). \)

2. Moreover, we can see that in this large deviation result the assumption of the concavity of a hazard function \( R_F \) can be removed.
For the proof of Theorem 3.2 we first need the next lemma.

**Lemma 3.3.** Assume that

\[ r := \limsup_{t \to \infty} \frac{tq_F(t)}{R_F(t)} < 1. \]

Then

\[ \int_{1/s}^{t} \exp(su) dF(u) \leq C < \infty. \]

**Proof.** Using the partial integration, we have

\[ \int_{1/s}^{t} \exp(su) dF(u) \leq s \int_{1/s}^{t} \exp(su) F(u) du + eF(1/s) = I + II. \]

Let us put \( r_\varepsilon = r + \varepsilon \), where \( \varepsilon \) is small enough and \( r_\varepsilon < 1 \). From the relation

\[ \limsup_{t \to \infty} \frac{tq_F(t)}{R_F(t)} < 1 \]

it follows that for \( u \) large enough

\[ (R_F(u)/u) = (uq_F(u) - R_F(u))/u^2 < -(1 - r_\varepsilon) R_F(u)/u^2 < 0, \]

so that \( R_F(t)/t \) is non-increasing. Then for \( u \) such that \( 1/s \leq u \leq t \) we obtain

\[ su - R_F(u) = \frac{R_F(t)u}{t} - R_F(u) \]

\[ \leq -(1 - r_\varepsilon) u \int_{u}^{t} (R_F(v)/v^2) dv \leq -(1 - r_\varepsilon) \frac{R_F(t)}{t^2} u(t - u). \]

Consequently, from (3.5) it follows that

\[ I = s \int_{1/s}^{t} \exp(su - R_F(u)) du \leq 4s/(1 - r_\varepsilon) s. \]

Moreover, we have

\[ II < \varepsilon. \]

The proof is complete.

**Proof of Theorem 3.2.** Let us define \( y \) as follows:

\[ y = \max \left\{ u > 0 : \frac{2\log u}{R_F(u)} \leq (1 - r_\varepsilon) \frac{t-u}{t} \right\}. \]

It is known that if \( r = 0 \), then \( y > \delta t \) for some \( \delta > 0 \). In the case \( r \neq 0 \) we can see that \( y > (1/2 + \delta_0) t \) for some \( \delta_0 > 0 \).
Let $\xi$ be the number of summands $X_k$, $k = 1, \ldots, n$, in $S_n$ such that $X_k \geq y$. Since the random variable $\xi$ has the Bernoulli distribution with parameters $n$ and $F(y)$, we may write

$$L_2 = \Pr(S_n > t, M_n \leq t) = \Pr(S_n > t, \xi = 0) + \Pr(S_n > t, \xi = 1, M_n \leq t) + \Pr(S_n > t, \xi \geq 2, M_n \leq t) = I + II + III.$$  

We have

$$III \leq \Pr(X_{n-1:m} > y, M_n \leq t) = O(1) \Pr^2(M_n > y) = O(1) \Pr(M_n > t) n \exp(-2R_F(y) + R_F(t)).$$

Under our assumptions we obtain

$$R_F(t) - R_F(y) \leq r_s(t-y)(t-y),$$

where $r_s$ is the same as in Lemma 3.3. Hence

$$R_F(t) - 2R_F(y) \leq -R_F(y) + r_s(t-y)(t-y) = -R_F(y) \left(1 - \frac{t-y}{y}\right).$$

Since $\varepsilon$ is an arbitrarily small positive quantity, in the case $r = 0$ we obtain

$$R_F(t) - 2R_F(y) \leq -R_F(y) + r_s(t-y)(t-y) \leq -R_F(\delta t)(1-\varepsilon) \leq -2\log t + O(1).$$

In the case $r \neq 0$ we have

$$R_F(t) - 2R_F(y) \leq -R_F(y) + r_s(t-y)(t-y) = -R_F(y) \left(1 - \frac{t-y}{y}\right) \leq -R_F(t/2)(1-r) \leq -2\log t + O(1).$$

Consequently, we obtain

$$III = O(1) \Pr(M_n > t) n/t^2 = o(1) \Pr(M_n > t) ns^2.$$  

Next we consider I. Let us define

$$V_k = \begin{cases} X_k & \text{for } X_k < y, \\ 0 & \text{for } X_k \geq y, \end{cases} \quad \sum_{k=1}^{n} V_k.$$  

Let $\delta_1, \delta_2, \ldots$ be a sequence of i.i.d. random variables with common d.f. $F$, which equals

$$F_s(u) = \min \left\{ 1, \left\{ \int_{-\infty}^{\delta_1} \exp(sv) dF(v) \right\} \left\{ \int_{-\infty}^{\delta_2} \exp(sv) dF(v) \right\}^{-1} \right\}.$$
Convergence in the precise large deviation theorem

So, to estimate the term $I$, we use the Cramer equality (see e.g. [9]): for any $u > 0$ we have

$$
P(S_n > u, \xi = 0) = \left( \mathbb{E}(\exp(sV_1)) \right)^n \int_{u}^{\infty} e^{-sv} d\mathbb{P} \left( \sum_{i=1}^{n} \delta_i < v \right).
$$

Hence

$$
P(S_n > u, \xi = 0) \leq \exp(-su) \left( \mathbb{E}(\exp(sV_1)) \right)^n \mathbb{P} \left( \sum_{j=1}^{n} \delta_j \geq u \right).
$$

We have

$$
\mathbb{E}\exp(sV_1) = \left( \int_{-\infty}^{y} + \int_{y}^{1/s} \right) \exp(su) dF(u) \leq J_1 + s \int_{1/s}^{y} \exp(su-R_F(u)) du := J_1 + sJ_2.
$$

Using the condition $\gamma > 2$, from Lemma 2.1 we get

$$
J_1 = 1 + O(1)s^2.
$$

Now we consider $J_2$. We have

$$
J_2 \leq s^2 \int_{1/s}^{y} u^2 \exp(su-R_F(u)) du.
$$

Let us define the function $Q_1$ as follows:

$$
Q_1(t) = R_F(t) - 2\log t, \quad t \geq t_1 \geq 1.
$$

Since $\lim_{t \to \infty} tq_F(t) > 2$, we infer that $Q_1$ is a hazard function. Let us put

$$
q_1(t) = \frac{d}{dt}Q_1(t), \quad t \geq t_1 \geq 1.
$$

We can show that under our assumptions

$$
\limsup_{t \to \infty} \frac{tq_1(t)}{Q_1(t)} = \limsup_{t \to \infty} \frac{tq(t) - 2}{R_F(t) - 2\log t} \leq \limsup_{t \to \infty} \frac{r_1(R_F(t) - 2\log t) + 2(r_1 \log t - 1)}{R_F(t) - 2\log t} < 1.
$$

We have

$$
\frac{R_F(t)}{t} = \frac{R_F(y) - 2\log y + 2\log y + R_F(t)}{y} - \frac{R_F(y)}{y} \leq s_1(y) + \frac{2\log y}{y} -(1-r_1) \frac{R_F(y)}{yt} (t-y) \leq s_1(y),
$$
where \( s_1 := s_1(y) = Q_1(y)/y \). Therefore, from Lemma 3.3 it follows that

\[
(3.7) \quad s \int_{y}^{\gamma} u^2 \exp (su - R(u)) \, du \leq s_1 \int_{y}^{\gamma} \exp (s_1 u - Q_1(u)) \, du < \infty.
\]

From (3.6) it follows that under our assumptions

\[
(3.8) \quad P(S_n \geq u, \xi = 0) \leq \exp (c^* n s^2) \exp (-su) P(\sum_{j=1}^{n} \delta_j \geq u).
\]

We have

\[
E\delta_1^2 \leq (E(\exp (s V_1)))^{-1} (\int_{1/s}^{1/s} u^2 e^{su} dF(u) + \int_{1/s}^{\gamma} u^2 e^{su} dF(u)).
\]

Since \( \gamma > 2 \), we obtain

\[
\int_{-\infty}^{1/s} u^2 e^{su} dF(u) < \infty.
\]

Note that

\[
\int_{1/s}^{\gamma} u^2 e^{su} dF(u) \leq es^{-2} \overline{F}(1/s) + s \int_{1/s}^{1/s} u^2 e^{su} \overline{F}(u) \, du + 2 \int_{1/s}^{\gamma} u e^{su} \overline{F}(u) \, du \\
\leq es^{-2} \overline{F}(1/s) + s \int_{1/s}^{1/s} u^2 e^{su} \overline{F}(u) \, du + 2s \int_{1/s}^{\gamma} u^2 e^{su} \overline{F}(u) \, du.
\]

Using (3.7), we obtain

\[
\int_{1/s}^{\gamma} u^2 e^{su} dF(u) < \infty.
\]

Hence \( E\delta_1^2 < \infty \). From this it follows that

\[
(3.9) \quad P(\sum_{i=1}^{n} \delta_i > t) \leq nE\delta_1^2/t^2 = O(1)n/t^2.
\]

Application of (3.9) now shows that

\[
I = O(1)P(M_n > t) \exp (c^* n s^2)/t^2.
\]

To complete the proof, it remains to estimate \( II \). For \( \sqrt{n \log ns} < 1 \) we have

\[
II = P(S_n > t, t \geq M_n > y, X_{n-1:n} \leq y)
\]

\[
= P(S_n > t, t - 1/s \geq M_n > y, X_{n-1:n} \leq y)
\]

\[
+ P(S_n > t, t - \sqrt{n \log n} \geq M_n > t - 1/s, X_{n-1:n} \leq y)
\]

\[
+ P(S_n > t, t \geq M_n > t - \sqrt{n \log n}, X_{n-1:n} \leq y) := A + B + C.
\]
Using (3.4), (3.8) and (3.9) we obtain

\[
A = O(1) n \int_y^{t-1/s} P(S_{n-1} \geq t-u, \max_{k \leq n-1} X_k < y) \, dF(u)
\]

\[
= O(1) n \int_y^{t-1/s} P(\sum_{i=1}^n \delta_i \geq t-u) \exp(-s(t-u)) \, dF(u)
\]

\[
= O(1) n P(\sum_{i=1}^n \delta_i \geq 1/s) \exp(-st) \int_y^{t-1/s} \exp(su) \, dF(u)
\]

\[
= O(1) P(M_n > t) P(\sum_{i=1}^n \delta_i \geq 1/s) = O(1) P(M_n > t)n s^2.
\]

Now, we use the next result of [5]: let \( Y_1, Y_2, \ldots \) be a sequence of i.i.d. random variables such that \( EY_1 = 0, E|Y_1|^\beta < \infty \), where \( \beta \geq 2 \). Let us put \( B_n = \sum_{k=1}^n EY_k^2 \), \( M_{\beta,n} = \sum_{k=1}^n E|Y_k|^\beta \). Then

\[
P(\sum_{k=1}^n Y_k \geq x) \leq (1 + 2/\beta)^\beta M_{\beta,n} x^{-\beta} + \exp(-c_0 x^2 B_n^{-1}).
\]

Moreover, we have

\[
B \leq n \int_y^{t-1/s} P(S_{n-1} \geq t-u, \max_{k \leq n-1} X_k < y) \, dF(u)
\]

\[
= O(1) n F(t) P(S_{n-1} \geq \sqrt{n \log n})
\]

\[
= O(1) P(M_n > t) P(S_{n-1} \geq \sqrt{n \log n}) = O(1) P(M_n > t)n^{1-\gamma/2}.
\]

For \( C \), we have

\[
C = P(S_n > t, t \geq M_n > t - \sqrt{n \log n}, X_{n-1,n} \leq y) = O(1) P(t \geq M_n > t - \sqrt{n \log n})
\]

\[
= O(1) \{ P(M_n > t - \sqrt{n \log n}) - P(M_n > t) \}
\]

\[
= O(1) P(M_n > t) \left( \int_{t-\sqrt{n \log n}}^t q_F(u) \, du - 1 \right)
\]

\[
= O(1) P(M_n > t) \left( \int_{t-\sqrt{n \log n}}^t q_F(u) \, du \right) = O(1) P(M_n > t) \sqrt{n \log ns}.
\]

If \( \sqrt{n \log ns} \geq 1 \), then

\[
II = P(S_n > t, t \geq M_n > y, X_{n-1,n} \leq y)
\]

\[
= O(1) n \int_y^{t} P(S_{n-1} \geq t-u, \max_{k \leq n-1} X_k < y) \, dF(u)
\]
Hence

$$II = O(1)P(M_n > t)(\sqrt{n \log ns + ns^2 + n^{1-\gamma/2}}).$$

The lower bound of $\Lambda_n(t)$ follows from Lemma 3.1 with $z = 1/\sqrt{n \log n}$. Thus Theorem 3.2 is proved.

4. EXAMPLE

We say that d.f. $F$ belongs to the class $\mathcal{D}$ of dominated-variation distributions if its tail $\bar{F}$ satisfies

$$\limsup_{t \to \infty} \frac{F(t)}{F(2t)} < \infty.$$

It follows from this definition that the class of distributions with regularly varying right tails is contained in $\mathcal{D} \cap \mathcal{Q}$.

It is well known (see e.g. [6]) that if $F \in \mathcal{D} \cap \mathcal{Q}$, then $F \in \mathcal{S}$.

It is also known ([7], Theorem 3.3) that if $\limsup_{t \to \infty} tq(t) < \infty$, then $F \in \mathcal{D} \cap \mathcal{Q}$. On the other hand, if the hazard rate $q$ is non-increasing, then the statements $F \in \mathcal{D} \cap \mathcal{Q}$ and $\limsup_{t \to \infty} tq(t) < \infty$ are equivalent (see [7], Corollary 3.4).

The next result is true.

**Corollary 4.1.** Assume that

(i) $E X_1 = 0$;
(ii) $A := \limsup_{t \to \infty} tq(t) < \infty$;
(iii) $\gamma > 2$.

Then for some $c_0 > 0$, $c^* > 0$, $C_1 > 0$, $C_2 > 0$

$$-P(M_n > t)(c_0 n^{1-\gamma/2} + A \sqrt{n \log n/t}) \leq \Lambda_n(t)$$

$$\leq P(M_n > t)(\exp(c^* ns^2)/t^2 + C_1 n^{1-\gamma/2} + C_2 \sqrt{n \log n/t}).$$

**Proof.** We restrict ourselves only to indicating the changes which are necessary to make in the proof of Theorem 3.2. The basic change is in the estimates of the term II.

For $t > \sqrt{n \log n}$ we have

$$II = P(S_n > t, t \geq M_n > y, X_{n-1:n} \leq y)$$

$$= P(S_n > t, t - \sqrt{n \log n} \geq M_n > y, X_{n-1:n} \leq y)$$

$$+ P(S_n > t, t \geq M_n > t - \sqrt{n \log n}, X_{n-1:n} \leq y) := A + B.$$
For $t$ large enough we have $y > \delta t$, where $\delta > 0$. We obtain
\[
A \leq n \int_{y}^{\infty} P(S_{n-1} \geq t-u, \max_{k<n-1} X_k < y) dF(u)
\]
\[
= O(1) nF(t) P(S_{n-1} \geq \sqrt{n \log n})
\]
\[
= O(1) P(M_n > t) P(S_{n-1} \geq \sqrt{n \log n}) = O(1) P(M_n > t) n^{1-\gamma/2}.
\]
For $t > \sqrt{n \log n}$ and $n$ large enough we have
\[
P(S_n > t, t \geq M_n > t - \sqrt{n \log n}, X_{n-1:n} \leq y) = O(1) P(t \geq M_n > t - \sqrt{n \log n})
\]
\[
= O(1)(P(M_n > t - \sqrt{n \log n}) - P(M_n > t))
\]
\[
= O(1) P(M_n > t) \left( \exp \left( \int_{t-\sqrt{n \log n}}^{t} q_F(u) du \right) - 1 \right)
\]
\[
= O(1) P(M_n > t) (1 - \sqrt{n \log n/t})^{-A} - 1 = O(1) P(M_n > t) \sqrt{n \log n/t}.
\]
The proof is complete.

Remark. Let $t_n, n \in N$, be a sequence such that
\[
\limsup_{n \to \infty} \sqrt{n R_F(t_n)/t_n} \leq \epsilon (dc^*)^{-1/2} < \infty,
\]
where $c^*$ is the same as in Corollary 4.1 and $\infty > d > \alpha$. Then we have
\[
\exp(c^* n s^2)/t^2 \leq t/t^2 = o(1) \quad \text{as } n \to \infty
\]
uniformly with respect to $t \in (t_n, \infty)$. Hence under the conditions of Corollary 4.1 we obtain
\[
\Lambda_n(t) = o(1) P(M_n > t) \quad \text{as } n \to \infty
\]
uniformly with respect to $t \in (t_n, \infty)$.

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