WEAK LIMITS AND INTEGRALS OF GAUSSIAN COVARIANCES IN BANACH SPACES

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Abstract: Let $E$ be a separable real Banach space not containing an isomorphic copy of $c_0$. Let $S$ be a subset of $L(E^*, E)$ with the property that each $Q \in S$ is the covariance of the centred Gaussian measure $\mu_Q$ on $E$. We show that the weak operator closure of $S$ consists of Gaussian covariances again, provided that

$$\sup_{Q \in S} \int_E \|x\|^2 d\mu_Q(x) < \infty.$$  

If in addition $E$ has type 2, the same conclusion holds for the weak operator closure of the convex hull of $S$. As an application, sufficient conditions are obtained for the integral of Gaussian covariance operators to be a Gaussian covariance. Analogues of these results are given for the class of $\gamma$-radonifying operators from a separable real Hilbert space $H$ into $E$.

2000 AMS Mathematics Subject Classification: Primary 28C20; Secondary: 35R15, 60B11, 60H05.

Key words and phrases: Gaussian Radon measure, covariance operator, $\gamma$-radonifying operator, Fatou lemma, type 2, cotype 2, weak operator topology.

The full text is available here.