PROBABILITY
AND
MATHEMATICAL STATISTICS
Vol. 3, Fasc. 2 (1984), pp. 127–141

ON THE CENTRAL LIMIT THEOREM IN BANACH SPACE c_0

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Abstract: In the paper the central limit theorem and the rates of convergence in this theorem in Banach space c_0 are considered. Let $\xi_i = \xi_i^{(1)}, ..., \xi_i^{(n)}, ...), \ i = l, 2, ...,$ be i.i.d. c_0-valued random variables with $E\xi_1 = 0$ and covariance matrix T. Let μ be a zero-mean Gaussian measure on c_0 with covariance matrix T,

$$F_n(A) = P\{n^{-1/2} \sum_{i=1}^{n} \xi_i \in A\}.$$

The main result of the paper can be formulated as follows: if $|\xi_1^{(j)}| < M_j = (\ln j)^{-1/2} a_j, j > j_0$, where $\{a_j\}$ is an arbitrary sequence of positive numbers tending to zero, then F_n converges weakly to μ. Moreover, if instead of a_j we take a slowly increasing sequence $(\ln_k j)^{1/2+\varepsilon}$, where $\ln_k x = \ln \ln_{k-1} x$ and $k \geq 2$ is an arbitrary integer, then it is possible to construct $\xi_i, \ i \geq 1$, failing the central limit theorem.

If $|\xi_1^{(j)}| < M\sigma_j, \ \sigma_j^2 = E(\xi_1^{(j)})^2 = (\ln j)^{-(1+\delta)}, \ j \geq 2, \ \delta > 0$, and T satisfies one additional condition, then we get the estimate

$$\sup_{r \geq 0} |F_n(||x|| < r) - \mu(||x|| < r) = O(n^{-1/2+\varepsilon}), \ \varepsilon > 0.$$

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2000 AMS Mathematics Subject Classification: Primary: -; Secondary: -;

Key words and phrases: -

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