ON THE CENTRAL LIMIT THEOREM IN BANACH SPACE $c_0$

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Abstract: In the paper the central limit theorem and the rates of convergence in this theorem in Banach space $c_0$ are considered. Let $\xi_i = \xi^{(1)}_i, \ldots, \xi^{(n)}_i$, $i = 1, 2, \ldots$, be i.i.d. $c_0$-valued random variables with $E\xi_1 = 0$ and covariance matrix $T$. Let $\mu$ be a zero-mean Gaussian measure on $c_0$ with covariance matrix $T$.

$$F_n(A) = P\{n^{-1/2} \sum_{i=1}^{n} \xi_i \in A\}.$$  

The main result of the paper can be formulated as follows: if $|\xi^{(j)}_1| < M_j = (\ln j)^{-1/2} a_j, j > j_0$, where $\{a_j\}$ is an arbitrary sequence of positive numbers tending to zero, then $F_n$ converges weakly to $\mu$. Moreover, if instead of $a_j$ we take a slowly increasing sequence $(\ln k j)^{1/2+\varepsilon}$, where $\ln k = \ln \ln_{k-1} x$ and $k \geq 2$ is an arbitrary integer, then it is possible to construct $\xi_i, i \geq 1$, failing the central limit theorem.

If $|\xi^{(j)}_1| < M\sigma_j, \sigma^2_j = E(\xi^{(j)}_1)^2 = (\ln j)^{-(1+\delta)}, j \geq 2, \delta > 0$, and $T$ satisfies one additional condition, then we get the estimate

$$\sup_{r \geq 0} |F_n(||x|| < r) - \mu(||x|| < r) = O(n^{-1/2+\varepsilon}), \varepsilon > 0.$$  

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