UPPER AND LOWER CLASS SEPARATING SEQUENCES FOR BROWNIAN MOTION WITH RANDOM ARGUMENT

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Abstract: Let $X = X_1, X_2, \ldots$ be a sequence of random variables, let $W$ be a Brownian motion independent of $X$ and let $Z_k = W(X_k)$. A numerical sequence $(t_k)$ will be called an upper (lower) class sequence for $\{Z_k\}$ if

$$P(Z_k > t_k \text{ for infinitely many } k) = 0 \text{ (or 1, respectively).}$$

At a first look one might be tempted to believe that a “separating line” $(t_k^0)$, say, between the upper and lower class sequences for $\{Z_k\}$ is directly related to the corresponding counterpart $(s_k^0)$ for the process $\{X_k\}$. For example, by using the law of the iterated logarithm for the Wiener process a functional relationship

$$t_k^0 = \sqrt{2s_k^0 \log \log s_k^0}$$

seems to be natural. If $X_k = |W_2(k)|$ for a second Brownian motion $W_2$ then we are dealing with an iterated Brownian motion, and it is known that the multiplicative constant $\sqrt{2}$ in (0.1) needs to be replaced by $2 \cdot 3^{-3/4}$, contradicting this simple argument.

We will study this phenomenon from a different angle by letting $\{X_k\}$ be an i.i.d. sequence. It turns out that the relationship between the separating sequences $(s_k^0)$ and $(t_k^0)$ in the above sense depends in an interesting way on the extreme value behavior of $\{X_k\}$.

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