HCM PROPERTY AND THE HALF-CAUCHY DISTRIBUTION

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Abstract: Let $Z_\alpha$ and $\tilde{Z}_\alpha$ be two independent positive $\alpha$-stable random variables. It is known that $(Z_\alpha/\tilde{Z}_\alpha)^\alpha$ is distributed as the positive branch of a Cauchy random variable with drift. We show that the density of the power transformation $(Z_\alpha/\tilde{Z}_\alpha)^\beta$ is hyperbolically completely monotone in the sense of Thorin and Bondesson if and only if $\alpha \leq 1/2$ and $|\beta| \geq \alpha/(1 - \alpha)$. This clarifies a conjecture of Bondesson (1992) on positive stable densities.

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The full text is available here