Abstract: Feller’s volume 2 shows how to use the Key Renewal Theorem to prove that in the limit $x \to \infty$, the renewal function $U(x)$ of a renewal process with non-arithmetic generic lifetime $X$ with finite mean $E(X) = 1/\lambda$ and second moment differs from its linear asymptote $\lambda x$ by the quantity $\frac{1}{2} \lambda^2 E(X^2)$. His first edition (1966) (but not the second in 1971) asserted that a similar approach would refine this asymptotic result when $X$ has finite higher order moments. The paper shows how higher order moments may justify drawing conclusions from a recurrence relation that exploits a general renewal equation and further appeal to the Key Renewal Theorem.

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