A TWO-PARAMETER EXTENSION OF URBANIK’S PRODUCT CONVOLUTION SEMIGROUP

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Abstract: We prove that \( s_n(a, b) = \frac{\Gamma(an + b)}{\Gamma(b)} \), \( n = 0, 1, \ldots \), is an infinitely divisible Stieltjes moment sequence for arbitrary \( a, b > 0 \). Its powers \( s_n(a, b)^c \), \( c > 0 \), are Stieltjes determinate if and only if \( ac \leq 2 \). The latter was conjectured in a paper by Lin (2019) in the case \( b = 1 \). We describe a product convolution semigroup \( \tau_c(a, b) \), \( c > 0 \), of probability measures on the positive half-line with densities \( e_c(a, b) \) and having the moments \( s_n(a, b)^c \). We determine the asymptotic behavior of \( e_c(a, b)(t) \) for \( t \to 0 \) and for \( t \to \infty \), and the latter implies the Stieltjes indeterminacy when \( ac > 2 \). The results extend the previous work of the author and López (2015) and lead to a convolution semigroup of probability densities \( (g_c(a, b)(x))_{c>0} \) on the real line. The special case \( (g_c(a, 1)(x))_{c>0} \) are the convolution roots of the Gumbel distribution with scale parameter \( a > 0 \). All the densities \( g_c(a, b)(x) \) lead to determinate Hamburger moment problems.

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