KARHUNEN–LOÈVE DECOMPOSITION OF GAUSSIAN MEASURES ON BANACH SPACES

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Abstract: The study of Gaussian measures on Banach spaces is of active interest both in pure and applied mathematics. In particular, the spectral theorem for self-adjoint compact operators on Hilbert spaces provides a canonical decomposition of Gaussian measures on Hilbert spaces, the so-called Karhunen–Loève expansion. In this paper, we extend this result to Gaussian measures on Banach spaces in a very similar and constructive manner. In some sense, this can also be seen as a generalization of the spectral theorem for covariance operators associated with Gaussian measures on Banach spaces. In the special case of the standard Wiener measure, this decomposition matches with Lévy–Ciesielski construction of Brownian motion.

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