PROBABILITY
AND
MATHEMATICAL STATISTICS
Vol. 7, Fasc. 1 (1986), pp. 27–58

LAW OF THE ITERATED LOGARITHM FOR SUBSEQUENCES

Allan Gut

Abstract: Let $\{S_n\}_{n=1}^{\infty}$ denote the partial sums of i.i.d. random variables with mean 0. The present paper investigates the quantity

$$\limsup_{k\to\infty} S_{n_k}/\sqrt{n_k \log\log n_k},$$

where $\{n_k\}_{k=1}^{\infty}$ is a strictly increasing subsequence of the positive integers. The first results are that if $EX_1^2 < \infty$, then the limit superior equals $\sigma\sqrt{2}$ a.s. for subsequences which increase "at most geometrically", and $\sigma\varepsilon^*$, where

$$\varepsilon^* = \inf\{\varepsilon > 0; \sum_k (\log n_k)^{-\varepsilon^2/2} < \infty\},$$

for subsequences which increase "at least geometrically". We also perform a refined analysis for the latter case and finally present criteria for the finiteness of

$$E\sup_k (S_{n_k}/\sqrt{n_k \log\log n_k})^2$$

in both cases.

2000 AMS Mathematics Subject Classification: Primary: -; Secondary: -;

Key words and phrases: -

THE FULL TEXT IS AVAILABLE HERE