LAW OF THE ITERATED LOGARITHM - CLUSTER POINTS OF DETERMINISTIC AND RANDOM SUBSEQUENCES

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Abstract: Let \( \{X_k\}_{k=1}^{\infty} \) be a sequence of i.i.d. random variables with mean 0 and finite, positive variance \( \sigma^2 \) and let

\[ S_n = \sum_{k=1}^{n} X_k, \quad n \geq 1. \]

Further, let

\[ \varepsilon^*\left(\{n_k\}\right) = \inf\{\varepsilon > 0; \sum_{k=3}^{\infty} (\log n_k)^{-\varepsilon^2/2} < \infty\}, \]

where \( \{n_k\}_{k=1}^{\infty} \) is a strictly increasing subsequence of the positive integers. Then the set of cluster points of \( \{S_{n_k}/\sqrt{n_k \log \log n_k}\}_{k=3}^{\infty} \) equals \( [-\sigma \sqrt{2}, \sigma \sqrt{2}] \) a.s. if \( \liminf_{k \to \infty} n_k/n_{k+1} > 0 \), and \( [-\sigma \varepsilon^*\left(\{n_k\}\right), \sigma \varepsilon^*\left(\{n_k\}\right)] \) a.s. if \( \limsup_{k \to \infty} n_k/n_{k+1} < 1 \).

These results are then applied to randomly indexed partial sums.

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