ON THE DOBRUSHIN’S HYPOTHESIS

BY

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Abstract. The central limit theorem for the stationary random processes under generalized mixing conditions is proved. The well-known Ibragimov’s results are given as a special case of the theorem received.

1. In [3] Dobrushin has introduced certain weak dependence conditions for random fields, which form natural generalization of the known mixing conditions to be found in [4]. In the same paper Dobrushin has suggested that under these generalized mixing conditions it is possible to prove a central limit theorem which would contain the well-known results as special cases. Here we prove the Dobrushin hypothesis for 1-dimensional case.

2. Let \( X \) be a metric space with metric \( d(x, \bar{x}) \), \( x, \bar{x} \in X \), \( \mathcal{B} \) be its Borel \( \sigma \)-algebra, and \( P \) and \( Q \) be the probability distribution on \( \mathcal{B} \).

The quantity \( R(P, Q) = \inf E d(\eta, \xi) \), where the “\( \inf \)” is taken over all 2-dimensional random vectors \((\eta, \xi)\) which marginal distributions coincide with \( P \) and \( Q \), respectively, is a metric on the space of probability distributions on \((X, \mathcal{B})\) and is called the Wasserstein or, sometimes, the Kantorovich-Rubinstein distance [6].

In [3] it is shown that if

\[
q(x, \bar{x}) = \begin{cases} 
1, & x \neq \bar{x}, \\
0, & x = \bar{x}, 
\end{cases}
\]

then

\[
R(P, Q) = \sup_{B \in \mathcal{B}} |P(B) - Q(B)|,
\]

i.e. \( R \) becomes the well-known variation metric.
If $X = R^k$, where $k$ is any positive integer and

$$q^{(k)}(x, \bar{x}) = \sum_{i=1}^{k} |x_i - \bar{x}_i|, \quad x, \bar{x} \in R^k,$$

then (cf. [5])

$$R(P, Q) = \int_{R^k} |F(x) - G(x)| \, dx,$$

where $F(x)$ and $G(x)$ are the distribution functions of $P$ and $Q$, respectively.

Let $\{\xi_t\} = \{\xi_t, t \in Z\}$ be a stationary process which takes values in the space $X$, where $Z$ is the set of the integers, and let $P = \{P_V, V \subset Z\}$ be the set of its finite-dimensional distributions. Here every $P_V$ is a probability measure on the $\sigma$-algebra of Borel subsets of the metric space $X^{|V|} = \{(x_1, \ldots, x_V), x_i \in X, i = 1, 2, \ldots, |V|\},$

$$q_V(x, \bar{x}) = \sum_{i=1}^{|V|} q(x_i, \bar{x}_i), \quad x, \bar{x} \in X^{|V|},$$

where $|V|$ denotes the number of points in a (finite) set $V$.

We say that a random process $\{\xi_t\}$ satisfies the generalized strong mixing condition (g.m.c.) if

$$(3) \quad R(P_{(-k,0)\cup(n,n+m)}, P_{(-k,0) \times P_{(n,n+m)}}) \leq \alpha_q(n) \quad \text{for any } k, m, n \in N,$$

where $\alpha_q(n) \to 0$ as $n \to \infty$. Here $(a, b)$ denotes the set of integers between $a$ and $b$, $a < b$ (a, b $\in Z$).

It is clear that if $X = R$ and the metric $\rho$ is discrete, i.e. coincides with (1), then (3) is the usual Rosenblatt strong mixing condition

$$(4) \quad |P(AB) - P(A) P(B)| \leq \alpha(n)$$

for any $A \in \sigma(\xi_t, t \leq 0)$ and $B \in \sigma(\xi_t, t \geq n)$, $n = 1, 2, \ldots$ and $\alpha(n) \to 0$ as $n \to \infty$.

By changing the space $X$ and the metric $\rho$ one can obtain various new mixing conditions. For instance, if $X = R$ and $q(x, \bar{x}) = |x - \bar{x}|, x, \bar{x} \in R$, then (3) reduces to

$$(5) \quad \left\{ \int_{R^{k+m}} |P(\bigcap_{t \in V_1, V_2} (\xi_t < x_t)) - P(\bigcap_{t \in V_1} (\xi_t < x_t)) P(\bigcap_{t \in V_2} (\xi_t < x_t))| \, dx_1 \cdots dx_k \right\} \times \prod_{t \in V_1 \cup V_2} dx_t \leq \hat{\alpha}(n),$$

where $V_1 = (-k, 0), V_2 = (n, n+m), \hat{\alpha}(n) \to 0$ as $n \to \infty$ independently of $k, m \in N$.

We will use mixing conditions (4) and (5) to illustrate our general proposition.
Note that various conditions under which the random field satisfies g.m.c. have been presented in [3].

3. Let $f(x), x \in X$, be a continuous function on $(X, \mathfrak{g})$ and let $\tau^f(\gamma)$, $\gamma \in \mathbb{R}_+$, denote the continuity modulus of $f$, i.e.

$$\tau^f(\gamma) = \sup_{(x, \overline{x}) \neq (x, \overline{x}) < \gamma} |f(x) - f(\overline{x})|.$$

We say that the process $\{\xi_t\}$ satisfies the central limit theorem (CLT) with function $f$ if, for any $s \in \mathbb{R}$,

$$\lim_{n \to \infty} P\left(\sqrt{\sigma^2} \sum_{i=1}^{n} f(\xi_i)^{-1/2} \sum_{i=1}^{n} (f(\xi_i) - E f(\xi_i)) < s\right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{s} e^{-u^2/2} du,$$

where $\sigma^2$ stands for the variance.

Our main result is as follows:

**Theorem.** Suppose that the stationary random process $\{\xi_t\}$ satisfies the g.m.c. and:

1. for some $\delta > 0$, $E |f(\xi_t)|^{2+\delta} < \infty$ (or, with probability 1, $|f(\xi_t)| < C < \infty$);

2. there exists a decreasing sequence $\gamma_n \in \mathbb{R}_+, \gamma_n \downarrow 0$ as $n \to \infty$, such that $\gamma_n^{-1} \alpha_\delta(n) \downarrow \beta(n)$, and

$$\sum_{n=1}^{\infty} \tau^f(\gamma_n) < \infty, \quad \sum_{n=1}^{\infty} \beta^{\delta(2+\delta)}(n) < \infty \quad (or \sum_{n=1}^{\infty} \beta(n) < \infty).$$

Then the series

$$\sigma^2 = E(f(\xi_t) - E f(\xi_t))^2 + 2 \sum_{n=2}^{\infty} E(f(\xi_t) - E f(\xi_t))(f(\xi_t) - E f(\xi_t))$$

converges and, if $\sigma^2 \neq 0$, then the process satisfies the CLT with the function $f$.

Note that, in case of $X = \mathbb{R}$ and discrete metric $\mathfrak{g}$, the continuity modulus $\tau^f(\gamma)$, $\gamma \in \mathbb{R}_+$, of any function $f$ on $X$ is equal to zero for $\gamma < 1$ and so the well-known Ibragimov's result [2] on CLT for stationary random processes becomes a special case of our theorem for $f(x) = x, x \in \mathbb{R}$. It has been shown in [5] and [6] that these results of Ibragimov practically cannot be improved.

If $X = \mathbb{R}$ and $\mathfrak{g}(x, \overline{x}) = |x - \overline{x}|, x, \overline{x} \in \mathbb{R}, f(x) = x$, then $\tau^f(\gamma) = \gamma, \gamma \in \mathbb{R}_+$, and we have the following

**Corollary.** Suppose that the stationary random process $\{\xi_t\}$ with values in $\mathbb{R}$ satisfies the mixing condition (5) and, for some $\delta > 0$, $E |\xi_t|^{2+\delta} < \infty$. If for some $\varepsilon > 0$ the series

$$\sum_{n=1}^{\infty} n^{1+\varepsilon} \alpha^{\delta(2+\delta)}(n) < \infty$$

If $\sigma^2 \neq 0$, then the process satisfies the CLT with $f(x) = x, x \in \mathbb{R}$. 

4. We state now some necessary estimates for the covariance of random variables.

**Lemma 1.** Suppose that a stationary random process \{ξ_t\} satisfies the g.m.c., \( V_1 = (-k, 0) \), \( V_2 = (n, n + m) \) and \( ξ_t = (ξ_i, t ∈ I), I ∈ \mathbb{Z} \). Let the functions \( φ_i(x, t ∊ V_i) \), \( i = 1, 2 \), be continuous with respect to the metric \( ϑ_ρ_i \), \( i = 1, 2, \) respectively, and \( τ_ρ_i(y), i = 1, 2, \) be their continuity moduli. Suppose also that, for some \( s, u > 1 \) \((1/s + 1/u < 1)\) the moments \( E|φ_1(ξ_i)|^s \) and \( E|φ_2(ξ_i)|^u \) exist. Then, for any \( γ > 0 \),

\[
(4') \quad \tau_ρ_i(y) E|φ_2(ξ_i)| + \tau_ρ_i(y) E|φ_1(ξ_i)| + \frac{2C_1 C_2 χ_ρ(n)}{γ}.
\]

If, with probability 1, \( |φ_i(ξ_i)| ≤ C_i < ∞, i = 1, 2, \) then the right-hand side of (4) may be replaced by

\[
(4) \quad \tau_ρ_i(y) E|φ_2(ξ_i)| + \tau_ρ_i(y) E|φ_1(ξ_i)| + \frac{2C_1 C_2 χ_ρ(n)}{γ}.
\]

**Proof.** Let \( φ(x, t ∊ V_1 ∪ V_2) = φ_1(x, t ∊ V_1) φ_2(x, t ∊ V_2) \). Suppose the random vector \( η_{V_1 ∪ V_2} = (η_i, t ∊ V_1 ∪ V_2) \) has the distribution \( P_{V_1} × P_{V_2}, P_{V_i} ∈ P, i = 1, 2, \)

\( A_γ = \{ η_{V_1 ∪ V_2}(ξ_i), η_{V_1 ∪ V_2} < γ \}, \)

\( \bar{A}_γ \) is the complement of \( A_γ \) and, with probability 1,

\[ |φ_i(ξ_i, t ∊ V_i)| ≤ C_i < ∞, \quad i = 1, 2. \]

Then

\[
|Eφ_1(ξ_i) φ_2(ξ_i)| ≤ |Eφ_1(ξ_i) φ_2(η_i)|
\]

\[
≤ |Eφ_1(ξ_i) φ_2(η_i) - Eφ_1(ξ_i) φ_2(η_i)| + |Eφ_1(ξ_i) φ_2(η_i) - Eφ_1(ξ_i) φ_2(η_i)|
\]

\[
≤ E_{A_γ} |φ_1(ξ_i) φ_2(η_i)| + E_{A_γ} |φ_1(ξ_i) φ_2(η_i) - φ_1(η_i)| + E_{A_γ} |φ_1(ξ_i) φ_2(η_i) - φ_1(η_i) φ_2(ξ_i)| + \frac{2C_1 C_2 χ_ρ(n)}{γ}
\]

\[
≤ \tau_ρ_i(y) E|φ_1(ξ_i)| + \tau_ρ_i(y) E|φ_2(ξ_i)| + \frac{2C_1 C_2 χ_ρ(n)}{γ}.
\]

Thus inequality (4') is proved \((1)\).

\((1)\) We acknowledge that the idea of this inequality should be attributed to Dobrushin (see inequality (3.8) in [3]; note that inequality (3.8) contains a misprint: \( γ \) and \( δ(γ) \) should be interchanged).
Let us prove now inequality (4). Let
\[ \phi_i^K(x) = \begin{cases} \phi_i(x) & \text{if } |\phi_i(x)| \leq K_i, \\ K_i & \text{if } \phi_i(x) > K_i, \\ -K_i & \text{if } \phi_i(x) < -K_i, \end{cases} \]

where \( K_i \in \mathbb{R}_+ \), \( \phi_i^K(x) = \phi_i(x) - \phi_i^{K_i}(x), \) for \( x \in X', i = 1, 2, \) and \( \tau_i^{K_i}(\gamma) \) be the continuity modulus of \( \phi_i^{K_i}(x) \). It is easy to see that
\[ \tau_i^{K_i}(\gamma) \leq \tau_{
u_i}(\gamma), \quad |\phi_i^{K_i}(x)| \leq K_i, \quad i = 1, 2. \]

Further,
\[
|E[\phi_1(\xi_{\nu_1}) \phi_2(\xi_{\nu_2})] - E[\phi_1(\xi_{\nu_1})]E[\phi_2(\xi_{\nu_2})]| \\
\leq E[|\phi_1^{K_1}(\xi_{\nu_1}) \phi_2^{K_2}(\xi_{\nu_2}) - \phi_1^{K_1}(\eta_{\nu_1}) \phi_2^{K_2}(\eta_{\nu_2})|] + E[|\phi_1^{K_1}(\xi_{\nu_1}) \phi_2^{K_2}(\xi_{\nu_2}) - \phi_1^{K_1}(\xi_{\nu_1}) \phi_2^{K_2}(\xi_{\nu_2}) + \\
+ E[|\phi_1^{K_1}(\xi_{\nu_1}) \phi_2^{K_2}(\xi_{\nu_2})|] + E[|\phi_1^{K_1}(\xi_{\nu_1}) \phi_2^{K_2}(\xi_{\nu_2})|] + \\
+ E[|\phi_1^{K_1}(\xi_{\nu_1})| \phi_2^{K_2}(\xi_{\nu_2})|] + E[|\phi_1^{K_1}(\xi_{\nu_1})| E[\phi_2^{K_2}(\xi_{\nu_2})]| + \\
+ E[|\phi_1^{K_1}(\xi_{\nu_1})| E[\phi_2^{K_2}(\xi_{\nu_2})]]].
\]

Now it is enough to put
\[ K_1 = \left( \frac{\gamma E[|\phi_1(\xi_{\nu_1})|^n]}{\alpha_0(n)} \right)^{1/s}, \quad K_2 = \left( \frac{\gamma E[|\phi_2(\xi_{\nu_2})|^n]}{\alpha_0(n)} \right)^{1/u} \]

and by proceeding in the same way as in [2] (§ 2, p. 390) one can prove (4).

In the sequel the following statement will be important:

**Lemma 2.** Let \( (\zeta_1, \zeta_2, \ldots, \zeta_n) \) be a vector such that
\[ |E \prod_{s=1}^{n} \zeta_s| < \infty, \quad i = 1, 2, \ldots, n-1; \quad |E \zeta_i| \leq 1, \quad i = 1, 2, \ldots, n. \]

Then
\[
|E \prod_{s=1}^{n} \zeta_s - E \prod_{s=1}^{n} E \zeta_s| \\
\leq \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} |E(\zeta_i - 1)(\zeta_j - 1) \prod_{s=j+1}^{n} \zeta_s - E(\zeta_i - 1) E(\zeta_j - 1) \prod_{s=j+1}^{n} \zeta_s|. \]

**Proof.** It is well-known ([2], § 4, p. 429) that, under the conditions of Lemma 2,
\[
|E \prod_{s=1}^{n} \zeta_s - E \prod_{s=1}^{n} E \zeta_s| \\
\leq \sum_{i=1}^{n-1} |E \zeta_i \prod_{s=i+1}^{n} \zeta_s - E \zeta_i E \prod_{s=i+1}^{n} \zeta_s|. \]
We can write
\[ |E_s \prod_{s=i+1}^{n} \zeta_s - E_s E \prod_{s=i+1}^{n} \zeta_s| \]
\[ = |E(\zeta_1 - 1) \zeta_{i+1} + \prod_{s=i+2}^{n} \zeta_s - E(\zeta_1 - 1)E \zeta_{i+1} \prod_{s=i+2}^{n} \zeta_s| \]
\[ \leq |E(\zeta_1 - 1)(\zeta_{i+1} - 1) \prod_{s=i+2}^{n} \zeta_s - E(\zeta_1 - 1)E(\zeta_{i+1} - 1) \prod_{s=i+2}^{n} \zeta_s| + \]
\[ + |E(\zeta_1 - 1) \zeta_{i+2} \prod_{s=i+3}^{n} \zeta_s - E(\zeta_1 - 1)E \zeta_{i+2} \prod_{s=i+3}^{n} \zeta_s| \]

Continuing this procedure we obtain

(7) \[ |E_s \prod_{s=i+1}^{n} \zeta_s - E_s E \prod_{s=i+1}^{n} \zeta_s| \]
\[ \leq \sum_{j=i+1}^{n} |E(\zeta_1 - 1)(\zeta_j - 1) \prod_{s=j+1}^{n} \zeta_s - E(\zeta_1 - 1)E(\zeta_j - 1) \prod_{s=j+1}^{n} \zeta_s| \]

Substituting (7) into (6) we get Lemma 2.

5. Now we are going to prove our theorem.

In the sequel \( p = p(n) \) and \( q = q(n), n \in \mathbb{N} \), denote the positive integer-valued functions.

**Lemma 3.** Let \( \{\eta_t\} \) be a real-valued stationary random process such that \( E\eta_t^2 < \infty \) and:

1. \( \mathcal{D} S_n \sim cn, n \to \infty \), where \( 0 < c < \infty \), \( S_n = \sum_{i=1}^{n} \eta_i \);

2. for any function \( p = p(n), p(n) \to \infty, p = o(n), n \to \infty \), there exists a function \( q = q(n), q(n) \to \infty \), \( q = o(p), n \to \infty \), such that, for every real \( t \),

\[ |E \prod_{j=1}^{k} \exp \{it\hat{S}_p^{(j)}\} - \prod_{j=1}^{k} E \exp \{it\hat{S}_p^{(j)}\}| \to 0, \quad n \to \infty, \]

where

\[ \hat{S}_p^{(j)} = (\mathcal{D} S_n)^{-1/2} S_p^{(j)}, \quad S_p^{(j)} = \sum_{s=j+1}^{j+p+(j-1)q} (\eta_s - E\eta_s), \quad j = 1, 2, \ldots, k \]

and \( k = k(n) = \lfloor n/(p+q) \rfloor \).

Then for this process the CLT with identity function \( f \) holds.

**Proof.** It is clear that there exists a function \( p = p(n) \) such that \( (\mathcal{D} S_p^{(1)})^{-1} |S_p^{(1)}|^2 dP \to 0 \) as \( n \to \infty \), integrating for \( |S_p^{(1)}| \geq \varepsilon \sqrt{\mathcal{D} S_n} \), where \( \varepsilon > 0, p(n) \to \infty, p = o(n), n \to \infty \).
Now, to complete the proof, it remains to apply the Bernstein method ([2], § 4, p. 426) for this \( p = p(n) \).

Thus, in order to prove our theorem it is sufficient to verify the conditions of Lemma 3 for the process \( \{ \eta_t \} = \{ f(\xi_t) \} \).

Let us verify condition 1. We have

\[
D \left( \sum_{t=1}^{n} f(\xi_t) \right) = \sum_{t,s=1}^{n} (E f(\xi_t) f(\xi_s) - E f(\xi_t) E f(\xi_s))
\]

\[= nE(f(\xi_1) - E f(\xi_1)) + 2 \sum_{t=2}^{n} (n-t+1) (E f(\xi_1) f(\xi_t) - E f(\xi_1) E f(\xi_t)) \]

and

\[
\lim_{n \to \infty} n^{-1} D \left( \sum_{t=1}^{n} f(\xi_t) \right) = E f(\xi_1)^2 - 2 \sum_{t=2}^{n} t (E f(\xi_1) f(\xi_t) - E f(\xi_1) E f(\xi_t)).
\]

By Lemma 1 we get

\[|E f(\xi_1) f(\xi_t) - E f(\xi_1) E f(\xi_t)| \leq 2C \tau(\gamma_t) + C \left( \frac{\alpha_{\phi}(t)}{\gamma_t} \right)^{\beta(2+\delta)}, \quad 0 < C < \infty,\]

hence

\[\sigma_f^2 \leq E f^2(\xi_1) + 2C \sum_{t=1}^{\infty} \tau(\gamma_t) + 2C \sum_{t=1}^{\infty} \beta^{2+\delta}(t).\]

The second summand in (8) vanishes as \( n \to \infty \) by the well-known Kronecker lemma.

It remains to check condition 2. Let

\[W_t(x) = \exp \left\{ itB \sum_{s=1}^{m} f(x_s) \right\} - 1, \quad m \in \mathbb{N}, \quad 0 < B < \infty, \quad x \in X^m,\]

\( X^m \) being a metric space with metric (2). Since

\[|W_t(x) - W_t(\tilde{x})| \leq B |t| \sum_{s=1}^{m} |f(x_s) - f(\tilde{x}_s)|, \quad x, \tilde{x} \in X^m,\]

we conclude that the continuity modulus of the function \( W_t(x) \) does not exceed \( B |t| \tau^f(\gamma) \), where \( \tau^f(\gamma) \) is the continuity modulus of \( f \). By Lemma 1
for \( j > r \) and \( s = u = 2 + \delta, \delta > 0 \), we have

\[
(9) \quad |E(\exp \{it\hat{S}_p^{(n)}\} - 1)\{\exp \{it\hat{S}_p^{(0)}\} - 1\} \prod_{s=j+1}^k \exp \{it\hat{S}_p^{(s)}\}| - E(\exp \{it\hat{S}_p^{(n)}\} - 1)E(\exp \{it\hat{S}_p^{(0)}\} - 1) \prod_{s=j+1}^k \exp \{it\hat{S}_p^{(s)}\}|
\]

\[
\leq B_1 \frac{|t|}{\sqrt{n}} \tau^f(\gamma_{\nu - 0q}) E|\exp \{it\hat{S}_p^{(1)}\} - 1| + B_2 E^{2(2+\delta)}|\exp \{it\hat{S}_p^{(1)}\} - 1|^{2+\delta} \left( \frac{\alpha_q((j-r)q)}{\gamma_{\nu - 0q}} \right)^{8(2+\delta)}
\]

\[
\leq B_3 \left[ \frac{|t|}{\sqrt{n}} \frac{p \sqrt{p}}{n} \frac{p}{n} \sum_{j=1}^\infty \tau^f(\gamma_{jq}) + \frac{p^2}{n} \sum_{j=1}^\infty \frac{\alpha_q((j-r)q)}{\gamma_{jq}} \right], \quad 0 < B_i < \infty, \ i = 1, 2, 3.
\]

By Lemma 2 and (9) we get

\[
|E \prod_{j=1}^k \exp \{it\hat{S}_p^{(n)}\} - \prod_{j=1}^k E \exp \{it\hat{S}_p^{(j)}\}|
\]

\[
\leq B_4 \left[ |t| \frac{p \sqrt{p}}{n} \frac{p}{n} \sum_{j=1}^\infty \tau^f(\gamma_{jq}) + p \sum_{j=1}^\infty \beta^{8(2+\delta)}(jq) \right].
\]

and then

\[
|E \prod_{j=1}^k \exp \{it\hat{S}_p^{(n)}\} - \prod_{j=1}^k E \exp \{it\hat{S}_p^{(j)}\} \leq B_4 \left[ |t| \sqrt{p} \sum_{j=1}^\infty \tau^f(\gamma_{jq}) + p \sum_{j=1}^\infty \beta^{8(2+\delta)}(jq) \right].
\]

The monotonicity of the members of this series implies

\[
\tau(\gamma_{jq}) \leq \frac{2}{q \sup_{j \geq q/2} \tau(\gamma_k)},
\]

\[
\beta^{8(2+\delta)}(jq) \leq \frac{2}{q \sup_{j \geq q/2} \beta^{8(2+\delta)}(k), \quad j = 1, 2, \ldots,
\]

hence

\[
\sum_{j=1}^\infty \tau(\gamma_{jq}) \leq \frac{2}{q \sup_{j \geq q/2} \tau(\gamma_j)}, \quad \sum_{j=1}^\infty \beta^{8(2+\delta)}(jq) \leq \frac{2}{q \sup_{j \geq q/2} \beta^{8(2+\delta)}(j)}.
\]

Finally,

\[
(10) \quad |E \prod_{j=1}^k \exp \{it\hat{S}_p^{(n)}\} - \prod_{j=1}^k E \exp \{it\hat{S}_p^{(j)}\}|
\]

\[
\leq B_4 |t| \frac{2 \sqrt{p}}{q} \sum_{j \geq q/2} \tau(\gamma_j) + \frac{2p}{q} \sum_{j \geq q/2} \beta^{8(2+\delta)}(j),
\]
as it is obvious that one can choose the function \( q(n) \to \infty, q = o(p), n \to \infty \), such that the right-hand side of (10) tends to zero as \( n \to \infty \).

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