Second class particles and random Young tableaux
joint work with Dan Romik

Piotr Śniady

Polish Academy of Sciences
and
University of Wrocław
\[ \Omega \text{ — set of infinite Young tableaux} \]
Young tableau as a growth process

\[ t = 0 \]
Young tableau as a growth process

t = 1
Young tableau as a growth process

$t = 2$
Young tableau as a growth process

$t = 3$

13 18 21 30 24 19
11 17 14 15 10
6 9 12 8
4 5 7
3 2 1

1 2 3
Young tableau as a growth process

\[ t = 4 \]
Young tableau as a growth process

t = 5
Young tableau and dynamics of particles

$t = 0$
Young tableau and dynamics of particles

\[ t = 0 \]
Young tableau and dynamics of particles

$t = 1$
Young tableau and dynamics of particles
Young tableau and dynamics of particles

$t = 1$
Young tableau and dynamics of particles
Young tableau and dynamics of particles

$t = 2$
Young tableau and dynamics of particles

$t = 2$
Young tableau and dynamics of particles

$t = 3$
Young tableau and dynamics of particles

\[ t = 3 \]
Young tableau and dynamics of particles

t = 3
Young tableau and dynamics of particles

\[ t = 4 \]
Young tableau and dynamics of particles

\[ t = 4 \]
Young tableau and dynamics of particles
Young tableau and dynamics of particles

$t = 5$
jeu de taquin and second class particles

$\begin{array}{cccccccc}
13 & 18 & 21 & 30 & 24 & 19 \\
11 & 17 & 14 & 15 & 10 \\
6 & 9 & 12 & 8 \\
4 & 5 & 7 \\
3 & 2 \\
\end{array}$

$t = 1$

*jeu de taquin trajectory* — rouge path of the gap
jue de taquin and second class particles

jue de taquin trajectory — rouge path of the gap
jeu de taquin and second class particles

$\begin{array}{cccccccc}
13 & 18 & 21 & 30 & 24 & 19 \\
11 & 17 & 14 & 15 & 10 \\
6 & 9 & 12 & 8 \\
4 & 5 & 7 \\
3 & & & \\
\end{array}$

$t = 2$

$\textit{jeu de taquin trajectory} — \text{rouge path of the gap}$
jeu de taquin and second class particles

jeu de taquin trajectory — rouge path of the gap
jeu de taquin and second class particles

$\mathbf{t} = 3$

jeu de taquin trajectory — rouge path of the gap
jeu de taquin and second class particles

$\mathbf{t} = 3$

$\textit{jeu de taquin trajectory} — \textit{rouge path of the gap}$
jeu de taquin and second class particles

$jeu \ de \ taquin \ trajectory \ — \ rouge \ path \ of \ the \ gap$
jeu de taquin and second class particles

jeu de taquin trajectory — rouge path of the gap
**Jeu de taquin and second class particles**

**Jeu de taquin trajectory** — rouge path of the gap
jeu de taquin and second class particles
jeu de taquin and second class particles
jeu de taquin and second class particles
jeu de taquin and second class particles
jeu de taquin and second class particles
jeu de taquin and second class particles
jeu de taquin and second class particles

$t = 3$
jeu de taquin and second class particles
jeu de taquin and second class particles
jeu de taquin and second class particles

t = 4
jeu de taquin and second class particles
jeu de taquin and second class particles

$t = 5$
second class particle

\[
\begin{array}{c}
\text{second class particle} = \text{jeu de taquin}
\end{array}
\]
jew de taquin — overview

original tableau $T$
## Jeu de taquin — overview

![Jeu de taquin grid]

### Outcome of slidings

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>18</td>
<td>29</td>
<td>30</td>
<td>24</td>
<td>19</td>
</tr>
<tr>
<td>11</td>
<td>17</td>
<td>21</td>
<td>15</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>14</td>
<td>12</td>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The outcome of slidings is shown in the shaded cells.
new tableau $J(T)$
Jeu de taquin — overview

Jeu de taquin applied to $T$ gives two pieces of information:

- trajectory $p(T)$ of jeu de taquin,
- the new tableau $J(T)$,
infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\mapsto \text{recording tableau } \in \Omega$

insertion tableau \hspace{1cm} recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K
infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau $\in \Omega$

insertion tableau

recording tableau

1. start from the first row,
infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\mapsto$ recording tableau $\in \Omega$

insertion tableau  

recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① start from the first row,
② insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,
infinite Robinson-Schensted-Knuth (RSK) map

\[ \text{infinite word} \xrightarrow{\text{RSK}} \text{recording tableau} \in \Omega \]

1. start from the first row,
2. insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,
infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau $\in \Omega$

insertion tableau

recording tableau

1

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① start from the first row,
② insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,
④ information about the new box into the recording tableau,
infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\mapsto$ recording tableau $\in \Omega$

insertion tableau

recording tableau

$\text{F O N D P X B Z U L G E A T W R S M Y V C J H Q I K}$

1. start from the first row,
2. insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,

4. information about the new box into the recording tableau,
infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau $\in \Omega$

insertion tableau

recording tableau

\[
\begin{array}{cccc}
\text{F} & \text{O} & \text{N} & \text{D} \\
\text{P} & \text{X} & \text{B} & \text{Z} \\
\text{U} & \text{L} & \text{G} & \text{E} \\
\text{A} & \text{T} & \text{W} & \text{R} \\
\text{S} & \text{M} & \text{Y} & \text{V} \\
\text{C} & \text{J} & \text{H} & \text{Q} \\
\end{array}
\]

1

1. start from the first row,
2. insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,
4. information about the new box into the recording tableau,
infinite Robinson-Schensted-Knuth (RSK) map

\[
\text{infinite word } \xrightarrow{\text{RSK}} \text{ recording tableau } \in \Omega
\]

1. start from the first row,
2. insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,
3. information about the new box into the recording tableau,
infinite Robinson-Schensted-Knuth (RSK) map

\[ \text{infinite word} \xrightarrow{\text{RSK}} \text{recording tableau} \in \Omega \]

\begin{align*}
\text{insertion tableau} & \quad & \text{recording tableau} \\
\begin{array}{|c|c|c|c|c|}
\hline
F & 0 & & & \\
\hline
\end{array} & \quad & \\
\begin{array}{|c|c|c|c|c|}
\hline
1 & & & & \\
\hline
\end{array}
\end{align*}

\text{F O N D P X B Z U L G E A T W R S M Y V C J H Q I K}

① start from the first row,
② insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,

④ information about the new box into the recording tableau,
infinite Robinson-Schensted-Knuth (RSK) map

\[ \text{infinite word } \overrightarrow{\text{RSK}} \rightarrow \text{recording tableau } \in \Omega \]

<table>
<thead>
<tr>
<th>F</th>
<th>0</th>
</tr>
</thead>
</table>

insertion tableau

| 1 | 2 |

recording tableau

\[ \text{F O N D P X B Z U L G E A T W R S M Y V C J H Q I K} \]

1. start from the first row,
2. insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,

4. information about the new box into the recording tableau,
infinite Robinson-Schensted-Knuth (RSK) map

infinite word $^\text{RSK} \mapsto$ recording tableau $\in \Omega$

<table>
<thead>
<tr>
<th>F</th>
<th>O</th>
</tr>
</thead>
</table>

| 1 | 2 |

insertion tableau | recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

1. start from the first row,
2. insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,

4. information about the new box into the recording tableau,
infinite Robinson-Schensted-Knuth (RSK) map

\[ \text{infinite word} \overset{\text{RSK}}{\rightarrow} \text{recording tableau} \in \Omega \]

<table>
<thead>
<tr>
<th>F</th>
<th>O</th>
</tr>
</thead>
</table>

\[ \text{insertion tableau} \rightarrow \text{recording tableau} \]

\[ \begin{array}{c}
\text{F O N D P X B Z U L G E A T W R S M Y V C J H Q I K}
\end{array} \]

1. start from the first row,
2. insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,
3. information about the new box into the recording tableau,
infinite Robinson-Schensted-Knuth (RSK) map

\[
\text{infinite word } \xrightarrow{\text{RSK}} \text{ recording tableau } \in \Omega
\]

insertion tableau

recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

1 2

① start from the first row,
② insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,
④ information about the new box into the recording tableau,
infinite Robinson-Schensted-Knuth (RSK) map

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F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

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infinite word $\xrightarrow{\text{RSK}}$ recording tableau $\in \Omega$

insertion tableau

recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① start from the first row,
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Infinite Robinson-Schensted-Knuth (RSK) map

Infinite word $\xrightarrow{\text{RSK}}$ recording tableau $\in \Omega$

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infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau $\in \Omega$

insertion tableau

recording tableau

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2. insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,
3. insert the bumped element into the next row,
4. information about the new box into the recording tableau,
infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau $\in \Omega$

<table>
<thead>
<tr>
<th>O</th>
<th>F</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

insertion tableau

<table>
<thead>
<tr>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>

recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

1. start from the first row,
2. insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,
3. insert the bumped element into the next row,
4. information about the new box into the recording tableau,
The infinite Robinson-Schensted-Knuth (RSK) map takes an infinite word and maps it to a recording tableau in \( \Omega \).

<table>
<thead>
<tr>
<th>Insertion Tableau</th>
<th>Recording Tableau</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insertion</td>
<td>Recording</td>
</tr>
<tr>
<td>F O N D P X B Z U L G E A T W R S M Y V C J H Q I K</td>
<td></td>
</tr>
</tbody>
</table>

1. Start from the first row,
2. Insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,
3. Insert the bumped element into the next row,
4. Insert information about the new box into the recording tableau,
infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau $\in \Omega$

insertion tableau

recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

1. start from the first row,
2. insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,
3. insert the bumped element into the next row,
4. information about the new box into the recording tableau,
infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau $\in \Omega$

insertion tableau                       recording tableau

<table>
<thead>
<tr>
<th>O</th>
<th>D</th>
<th>N</th>
</tr>
</thead>
</table>

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

1. start from the first row,
2. insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,
3. insert the bumped element into the next row,
4. information about the new box into the recording tableau,
infinite Robinson-Schensted-Knuth (RSK) map

\[ \text{infinite word } \xrightarrow{\text{RSK}} \text{ recording tableau } \in \Omega \]

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

**infinite word**

**recording tableau**

**insertion tableau**

**recording tableau**

| F | O | N | D | P | X | B | Z | U | L | G | E | A | T | W | R | S | M | Y | V | C | J | H | Q | I | K |

1. **start from the first row,**
2. **insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,**
3. **insert the bumped element into the next row,**
4. **information about the new box into the recording tableau,**
infinite Robinson-Schensted-Knuth (RSK) map

\[ \text{infinite word } \begin{array}{c} \text{RSK} \\ \rightarrow \end{array} \begin{array}{c} \text{recording tableau} \end{array} \in \Omega \]

\[
\begin{array}{|c|c|c|} 
\hline
\text{F} & \text{D} & \text{N} \\
\hline
\end{array}
\quad
\begin{array}{|c|c|} 
\hline
3 & 1 \\
1 & 2 \\
\hline
\end{array}
\]

\begin{itemize}
\item \textbf{1} start from the first row,
\item \textbf{2} insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,
\item \textbf{3} insert the bumped element into the next row,
\item \textbf{4} information about the new box into the recording tableau,
\end{itemize}
infinite Robinson-Schensted-Knuth (RSK) map

infinite word $^{\text{RSK}} \mapsto$ recording tableau $\in \Omega$

insertion tableau  

<table>
<thead>
<tr>
<th>F</th>
<th>D</th>
<th>N</th>
</tr>
</thead>
</table>

recording tableau  

<table>
<thead>
<tr>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

start from the first row,
insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,
insert the bumped element into the next row,
information about the new box into the recording tableau,
Infinite Robinson-Schensted-Knuth (RSK) map

\[ \text{infinite word } \overset{\text{RSK}}{\mapsto} \text{ recording tableau } \in \Omega \]

\[ \begin{array}{cccc}
F & D & & \\
& & & \\
\end{array} \quad \begin{array}{cccc}
& & & \\
3 & & & \\
1 & 2 & & \\
\end{array} \]

**insertion tableau** \hspace{1cm} **recording tableau**

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

1. **start from the first row,**
2. **insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,**
3. **insert the bumped element into the next row,**
4. **information about the new box into the recording tableau,**
Infinite Robinson-Schensted-Knuth (RSK) map

Infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau $\in \Omega$

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4. Information about the new box into the recording tableau,
infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau $\in \Omega$

<table>
<thead>
<tr>
<th>O</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>N</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

insertion tableau  

<table>
<thead>
<tr>
<th>4</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

1. start from the first row,
2. insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,
3. insert the bumped element into the next row,
4. information about the new box into the recording tableau,
The infinite Robinson-Schensted-Knuth (RSK) map takes an infinite word and maps it to a pair of tableaux. The first tableau, the insertion tableau, is constructed by inserting letters as far to the right as possible, ensuring that the row is increasing and no gaps are created. The second tableau, the recording tableau, records information about the new boxes inserted. Here's how it works:

1. **Start from the first row**, insert the letter as far to the right as possible, so that the row is increasing and no gaps are created.
2. **Insert the bumped element into the next row**.
3. **Insert the bumped element into the next row**.
4. **Information about the new box into the recording tableau**.
infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\xrightarrow{\text{RSK}}$ recording tableau $\in \Omega$

<table>
<thead>
<tr>
<th>F</th>
<th>L</th>
<th>P</th>
<th>W</th>
<th>7</th>
<th>11</th>
<th>22</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>G</td>
<td>M</td>
<td>R</td>
<td>4</td>
<td>10</td>
<td>16</td>
<td>17</td>
</tr>
<tr>
<td>B</td>
<td>E</td>
<td>J</td>
<td>Q</td>
<td>3</td>
<td>9</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>A</td>
<td>C</td>
<td>H</td>
<td>I</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

insertion tableau

recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① start from the first row,
② insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,
③ insert the bumped element into the next row,
④ information about the new box into the recording tableau,
infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau $\in \Omega$

\[
\begin{array}{cccc}
7 & 11 & 22 & 24 \\
4 & 10 & 16 & 17 \\
3 & 9 & 14 & 15 \\
1 & 2 & 5 & 6 \\
\end{array}
\]

**insertion tableau**

**recording tableau**

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K
infinite Robinson-Schensted-Knuth (RSK) map

infinite word \( \overset{\text{RSK}}{\mapsto} \) recording tableau \( \in \Omega \)

\[
\begin{array}{cccc}
7 & 11 & 22 & 24 \\
4 & 10 & 16 & 17 \\
3 & 9 & 14 & 15 \\
1 & 2 & 5 & 6
\end{array}
\]

insertion tableau recording tableau

if \( X_0, X_1, \ldots \) are i.i.d. \( U(0, 1) \) random variables then

\[
\text{RSK}(X_0, X_1, \ldots) \overset{\text{distribution}}{=} \text{Plancherel measure}
\]
trajectories

jeu de taquin

second class particle
trajectories

\[ \Theta(T) \]

jeu de taquin

\[ \Theta(T) \] — asymptotic angle of jeu de taquin

second class particle

asymptotic speed of second class particle
The jeu de taquin dynamical system is isomorphic to i.i.d. shift dynamical system. The inverse map is given by $x_i = f^{-1}(\theta_i)$. The diagram shows the relationship between $T_0$, $J$, and $T_1$. 
The jeu de taquin dynamical system is isomorphic to i.i.d. shift dynamical system.

The inverse map is given by $x_i = f^{-1}(\theta_i)$.
the jeu de taquin dynamical system is isomorphic to i.i.d. shift dynamical system \((\Omega, \text{Plancherel}, J)\)
The jeu de taquin dynamical system $(\Omega, \text{Plancherel}, J)$ is isomorphic to the i.i.d. shift dynamical system $(\{0,1\}^N, \lambda, \text{equis})$. The inverse map is given by $x_i = f^{-1}(\theta_i)$. 

The diagram illustrates the relationship between the RSK algorithm and the dynamic systems $T_0, T_1, \ldots$.
jeu de taquin dynamical system \((\Omega, \text{Plancherel}, J)\)
The jeu de taquin dynamical system is isomorphic to i.i.d. shift dynamical system. The inverse map is given by

\[ x_i = f^{-1}(\theta_i) \]
i.i.d. shift dynamical system \( ([0, 1]^\mathbb{N}, \prod \text{Lebesgue}, s) \)

\[
\begin{align*}
(x_0, x_1, \ldots) \quad &\xrightarrow{s} \quad (x_1, x_2, \ldots) \quad &\xrightarrow{s} \quad &\ldots \\
\downarrow \text{RSK} \quad &
\downarrow J \quad &
\downarrow \text{RSK} \quad &
\downarrow J \quad &
\downarrow \ldots \\
T_0 \quad &J \quad &T_1 \quad &J \quad &\ldots \\
\downarrow \Theta \quad &
\downarrow \Theta \quad &
\downarrow \Theta \quad &
\downarrow \ldots \\
\theta_0 \quad &
\theta_1 \quad &
\theta_1 \quad &
\theta_1 \quad &
\ldots \\
\end{align*}
\]

jeu de taquin dynamical system \( (\Omega, \text{Plancherel}, J) \)
i.i.d. shift dynamical system $([0, 1]^\mathbb{N}, \prod \text{Lebesgue}, s)$

\[ (x_0, x_1, \ldots) \xrightarrow{s} (x_1, x_2, \ldots) \xrightarrow{s} \ldots \]

RSK \quad \text{inverse?} \quad \text{RSK}

\[ T_0 \quad J \quad T_1 \quad J \quad \ldots \]

\[ \Theta \quad \Theta \]

\[ \theta_0 \quad \theta_1 \]

jeu de taquin dynamical system $(\Omega, \text{Plancherel}, J)$
i.i.d. shift dynamical system \(([0, 1]^\mathbb{N}, \prod \text{Lebesgue}, s)\)

\[
\begin{align*}
(x_0, x_1, \ldots) & \overset{s}{\longrightarrow} (x_1, x_2, \ldots) & \overset{s}{\longrightarrow} & \ldots \\
\uparrow \text{RSK} & \quad \text{inverse!} & \downarrow \text{RSK} & \quad \Theta \\
T_0 & \quad J & T_1 & \quad J \\
\downarrow \Theta & \quad \Theta & \downarrow & \theta_0 \quad \theta_1 \\
\theta_0 & \quad \theta_1 & \ldots \\
\end{align*}
\]

jeu de taquin dynamical system \((\Omega, \text{Plancherel}, J)\)
i.i.d. shift dynamical system \([0, 1]^\mathbb{N}, \prod \text{Lebesgue}, s\)

\[
\begin{align*}
(x_0, x_1, \ldots) & \xrightarrow{s} (x_1, x_2, \ldots) & \xrightarrow{s} & \ldots \\
\Theta & \downarrow & RSK & \downarrow RSK \\
\theta_0 = f(x_0) & \uparrow \text{inverse!} & T_0 & \downarrow T_1 \\
\theta_1 = f(x_1) & \downarrow & \Theta & \downarrow \Theta
\end{align*}
\]

jue de taquin dynamical system \((\Omega, \text{Plancherel}, J)\)
i.i.d. shift dynamical system \((\mathbb{R}^N, \prod \text{Lebesgue}, s)\)

\[
\begin{align*}
(x_0, x_1, \ldots) & \overset{s}{\longrightarrow} (x_1, x_2, \ldots) & \overset{s}{\longrightarrow} & \ldots \\
\downarrow \text{RSK} & & \uparrow \text{inverse!} & & \downarrow \text{RSK} \\
T_0 & \overset{J}{\longrightarrow} & T_1 & \overset{J}{\longrightarrow} & \ldots \\
\downarrow \Theta & & \downarrow \Theta & & \\
\theta_0 = f(x_0) & & \theta_1 = f(x_1) & & \\
\end{align*}
\]

jeu de taquin dynamical system \((\Omega, \text{Plancherel}, J)\)

the jeu de taquin dynamical system is isomorphic to i.i.d. shift

the inverse map is given by \(x_i = f^{-1}(\theta_i)\)
main results

- slope angles $\theta_0, \theta_1, \ldots$
  (and hence asymptotic speeds of second class particles!)
  exist almost surely,
- they are independent random variables with explicit
distribution,
- RSK is an isomorphism between the dynamical system of shift
and jeu de taquin,
- jeu de taquin is an ergodic transformation,
open problems

Dan Romik, Piotr Śniady
Jeu de taquin dynamics on infinite Young tableaux and second class particles
arXiv:1111.0575