

1. Define $s_{x:\bar{n}} = \frac{a_{x:\bar{n}}}{n E_x}$. Explain the meaning of actuarial accumulation of $s_{x:\bar{n}}$ after demonstration of the following formulas Thus show

$$\begin{aligned}s_{x:\bar{n}} &= (1+i)^n \frac{1}{l_{[x]+n}} \sum_{k=1}^n l_{[x]+k} (1+i)^{n-k} \\ &= \sum_{k=1}^n \frac{1}{n-k E_{[x]+k}}.\end{aligned}$$

Similarly analyse $\ddot{s}_{x:\bar{n}} = \frac{\ddot{a}_{x:\bar{n}}}{n E_x}$

$$\begin{aligned}s_{x:\bar{n}} &= (1+i)^n \frac{1}{l_{[x]+n}} \sum_{k=0}^{n-1} l_{[x]+k} (1+i)^{n-k} \\ &= \sum_{k=0}^{n-1} \frac{1}{n-k E_{[x]+k}}.\end{aligned}$$

2. Show the retrospective formula for the whole life assurance:

$${}_k V_x^{\text{retro}} = P_x \ddot{s}_{x:\bar{k}} - \frac{A_1}{k E_x}.$$

Solution: Starting from the prospective formula we have

$$\begin{aligned}{}_k V_x &= A_{x+k} - P_x \ddot{a}_{x+k} \\ &= A_{x+k} - P_x \ddot{a}_{x+k} - P_x \ddot{s}_{x:\bar{n}} + P_x \ddot{s}_{x:\bar{n}}.\end{aligned}\tag{0.1}$$

We will need (showed it!)

$$\ddot{a}_{\bar{x}} = {}_k | \ddot{a}_x + \ddot{a}_{\bar{x+k}}, \quad \ddot{s}_{x:\bar{n}} = \frac{\ddot{a}_{k:\bar{x}}}{k E_x}, \quad {}_k | \ddot{a}_{\bar{x}} = {}_k E_x \ddot{a}_{x+k}.$$

Now the RHS of (0.1) is equal to

$$\begin{aligned}
& A_{x+k} + P_x \ddot{s}_{x:\bar{n}} - P_x (\ddot{a}_{x+k} + \frac{\ddot{a}_{k:\bar{x}}}{_k E_x}) \\
&= A_{x+k} + P_x \ddot{s}_{x:\bar{n}} - \frac{P_x}{_k E_x} (_k E_x \ddot{a}_{x+k} + \ddot{a}_{k:\bar{x}}) \\
&= A_{x+k} + P_x \ddot{s}_{x:\bar{n}} - \frac{P_x}{_k E_x} (_k | \ddot{a}_x + \ddot{a}_{k:\bar{x}}) \\
&= P_x \ddot{s}_{x:\bar{n}} + A_{x+k} - \frac{P_x}{_k E_x} \ddot{a}_x \\
&= P_x \ddot{s}_{x:\bar{n}} + A_{x+k} - \frac{A_x}{_k E_x} \\
&= P_x \ddot{s}_{x:\bar{n}} - \frac{A_1}{_k E_x},
\end{aligned}$$

where in the last equation we used

$$A_x = A_{1_{x:\bar{k}}} + {}_k E_x A_{x+k}.$$

3. Show the retrospective formula for the term life insurance:

$${}_k V_{1_{x:\bar{n}}}^{\text{retro}} = P_{1_{x:\bar{n}}} \ddot{s}_{x:\bar{k}} - \frac{A_1}{_k E_x},$$

for $0 \leq k \leq n$. Hint. Use the idea of Exercise 2. Show first the identity:

$$A_{1_{n:\bar{x}}} = A_{1_{x:\bar{k}}} + {}_x E_k A_{1_{x+k:n-\bar{k}}}.$$

4. Show the retrospective formula for continuous reserve:

$${}_t \bar{V}(\bar{A}_{1_{x:\bar{n}}}) = \bar{P}(\bar{A}_{1_{x:\bar{n}}}) \bar{s}_{t:\bar{x}} - \frac{\bar{A}_1}{_t E_x},$$

where $\bar{s}_{x:\bar{k}} = \frac{\bar{a}_{x:\bar{k}}}{_t E_x}$ and where $0 \leq t \leq n$.