

Axioms of Euclidean Geometry after David Hilbert, 1899
(slightly modified)

Primitive notions (or basic terms):

1. *point*,
2. *line* (i.e. straight line),
3. *incidence* relation (for pairs point–line), denoted with the symbol \in ,
4. relation of *order* for points from any line p , denoted with the symbol $<_p$,
5. *measure of segments*, denoted m ,
6. *measure of angles*, denoted with the symbol μ .

The other notions appearing in the statements of the axioms (such as segment, angle, half-line and half-plane) need to be defined, in terms of primitive notions. Here are their definitions:

- o for distinct points A, B belonging (i.e. incident) to a line p , the *interval* AB is a set consisting of points A and B and all points C such that $A <_p C <_p B$ or $B <_p C <_p A$;
- o a *half-line* started at A is any set of form $\{A\} \cup \{X \in p : A <_p X\}$ or $\{A\} \cup \{X \in p : X <_p A\}$, where p is any line containing A ;
- o an *angle* is a collection of two half-lines with common starting point not contained in a common line;
- o a *half-plane* bounded by a line p is any set of form $\{Y : Y \in p\} \cup \{C\} \cup \{X : CX \cap p = \emptyset\}$, where C is any point not lying on p .

Incidence axioms (i.e axioms concerning the relation of incidence)

- I1. For any two distinct points A and B there is exactly one line p passing through A and B .
- I2. Each line contains at least 2 distinct points.
- I3. There exist three points which do not lie on a common line.

Axioms of order

- P1. For points of any line p the relation $<_p$ is a linear order, i.e.:
 - (a) if $A <_p B$ then $A \neq B$;
 - (b) if $A \in p$, $B \in p$ and $A \neq B$, then precisely one of the conditions $A <_p B$, $B <_p A$ holds;
 - (c) if $A <_p B$ and $B <_p C$ then $A <_p C$.
- P2. (Moritz Pasch' Axiom) For any non-collinear points A, B, C and any line p not passing through any of these points, if p intersects the segment AB then it also intersects precisely one of the segments BC and AC .

Axioms of measure of segments

- M1. For any segment AB its measure $m(AB)$ is a positive real number.
- M2. For each half-line r started at a point A , and for any positive real number d there is a point $B \in r$ such that $m(AB) = d$.
- M3. If $A <_p B <_p C$ then $m(AB) + m(BC) = m(AC)$.

Axioms of measure of angles

- K1. For any angle rs (composed of half-lines r and s having common starting point) the measure $\mu(rs)$ is a real number from the open interval $(0, \pi)$.
- K2. For any line p and any half-plane Ω bounded by p , for any half-line r contained in p and any real number $\alpha \in (0, \pi)$, there is a half-line s contained in Ω which forms together with r an angle such that $\mu(rs) = \alpha$.
- K3. Let A, B, C and A', B', C' be two triples of non-collinear points. If $m(AB) = m(A'B')$, $m(AC) = m(A'C')$ and $\mu(BAC) = \mu(B'A'C')$ then $\mu(ABC) = \mu(A'B'C')$.

Parallel Axiom

- R. If A is a point not lying on a line p , then there is exactly one line through A not intersecting p .