Assume μ is a σ-finite measure on X. Let $T : L^2(X,\mu) \to L^2(X,\mu)$ act by $(Tf)(x) = \phi(x)f(x)$, for $\varphi \in L^\infty(X,\mu)$. Then $\varphi(x) \neq 0$ μ-a.e. For $\varphi(x) = a(x) + ib(x)$ we have $(Af)(x) = a(x)f(x)$, $(Bf)(x) = b(x)f(x)$. We claim there is $0 < r < 1$ such that the operator $A + rB$ is injective. Assume by contradiction that for any $0 < r < 1$ the operator $A + rB$ is not injective. As $(A + rB)(f)(x) = [a(x) + rb(x)]f(x)$, that means the set

$$X_r = \{x \in X \ \ a(x) + rb(x) = 0\}$$

has a positive measure. The sets X_r are pairwise disjoint, as $a(x) + ib(x) \neq 0$, a.e. For each n the set $A_n = \{r \in (0,1) : \mu(X_r) \geqslant \frac{1}{n}\}$ is finite. Therefore the set $A = \{r \in (0,1) : \mu(X_r) > 0\}$ is at most countable, which leads to a contradiction.