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TADEK PYTLIK IN MY MEMORIES

BY

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I met Tadeusz (Tadek for all of us) Pytlik in 1975. At that time I was a freshman at the Mathematical Institute of University of Wrocław and Tadek was a young lecturer with a recent PhD. He held calculus classes. Occasionally he substituted for Czesław Ryll-Nardzewski, who lectured in analysis during the first semester. I remember Tadek as a smart and handsome man, looking healthy and fit. When Ryll-Nardzewski left our university Tadek took over the analysis course and taught our group until the end of our third year of studies. Already at that time he was considered one of the best lecturers. He was known for beautiful handwriting. My best notes from the university years were those in analysis. He coauthored with Janusz Górniak a "Problem Book in Functional Analysis" published by the Technical University of Wrocław in 1976. I admired that book and actually studied the topic by solving all problems in it. The book has been in use in our department, as well as at the Technical University, for 25 years now. Later on, together with Marek Bożejko, they taught us various special topics in functional and harmonic analysis, and greatly influenced students from our group. Most of those students, including myself, decided to work in analysis and to prepare MSc theses under their supervision.

After obtaining my MSc in 1980 I got a position as a teaching assistant. I was supposed to teach and to work on my PhD thesis. Tadek submitted his habilitation in 1980, defended it a year later, and went to the University of Bielefeld for one year at the invitation of Horst Leptin. That's why Marek Bożejko took care of me during my first year at the institute. It was Marek who brought to Wrocław the discrete harmonic analysis, free groups etc. Already one of the main papers in Tadek's habilitation thesis [12] concerned harmonic analysis on free groups. There he reproved in a simple way some of the previously known results on the spectrum of the algebra of radial functions on the free group on $\ell^1(G)$ and $\ell^2(G)$. He showed new results concerning the spectrum of this algebra in $\ell^p(G)$. He also proved that the algebra of radial functions is maximal abelian in the von Neumann algebra of the group. This allowed him to obtain a new decomposition of the left regular representation of the free group into irreducible components. This paper is still being cited since people keep considering the problem of maximal commutativity of certain subalgebras. In the Fall of 1981 Tadek came back from Bielefeld. At that time Marek gave me the problem whether weak type translation invariant operators on free group are automatically of strong type on $\ell^2(G)$. I was able to solve it in the negative for $p = 2$. Next, Marek asked me to do the same for $\ell^p(G)$ with $p > 2$, since no infinite group was known for which this problem has been resolved. My method for $p = 2$ was to estimate the strong operator norm and the weak operator norm of the indicator functions of the elements in the free group of length n. It turned out that these norms were inequivalent on this particular sequence of functions. Unfortunately, according to Tadek's work [12] these norms are obviously equivalent in $\ell^p(G)$, due to a precise formula for the strong operator norm given in [12]. However, Tadek didn't provide the proof. The formula was just a remark made after one of the results. Actually, it yielded the spectral radius of the convolution operator with χ_n, which may be smaller than the operator norm in ℓ^p. I was hoping that the norm was actually greater and inquired Tadek about the proof. This was the origin of Tadek's beautiful paper [15] where he showed that the claim in his Crelles paper [12] was correct. The proof is really smart and relies on a tricky decomposition of the Poisson kernel associated with a simple random walk on G.

About the same time Tadek wrote another paper [17] where he constructed convolution operators on free groups which are bounded on $\ell^{p_0}(G)$ and unbounded on any $\ell^p(G)$ for $p \neq p_0$. This is a really nice piece of mathematics containing direct but ingenious computations.

I had luck to have also Marek around. At that time he was like a volcano, bursting every now and then with open problems for young people. A problem he posed to me was to estimate the so called completely bounded norm of the multiplier operators with χ_n and decide whether this norm can be estimated independently of the number of generators. This was shown by Michael Leinert for $n = 1$. I was able to get the estimate of $2n$ and at the same time I got a one-line proof of Leinert's result. Still the general case of $n \geq 1$ took me two hours of a seminar to show all the details. Tadek was in the audience and the next week he showed me a very simple proof of my result. His reasoning based on the following fact. Assume a function $m : G \to \mathbb{C}$ is of the form

$$m(y^{-1}x) = \langle a(x), b(y) \rangle,$$

where $a, b : G \to \mathcal{H}$ for some Hilbert space $\mathcal{H}$. Assume

$$A = \sup_x \|a(x)\|, \qquad B = \sup_y \|b(y)\|.$$

Then it is well known that $\|m\|_{cb} \le AB$. Now consider the free group G with a fixed set of free generators. Every nontrivial element x in G can be represented uniquely as a reduced word whose letters are generators and their inverses. If $x \neq e$ then $x = \overline{x}u$ where u is the last letter of x. Define a mapping P on $\ell^2(G)$ by

$$P\delta_x = \begin{cases} \delta_{\overline{x}}, & x \neq e, \\ 0, & x = e. \end{cases}$$

Let mappings a and b from G into the Hilbert space $\ell^2(G)^{n+1}$ act by the rule

$$a(x) = (\delta_x, P\delta_x, \ldots, P^{n-1}\delta_x, P^n\delta_x),$$
$$b(y) = (P^n\delta_y, P^{n-1}\delta_y, \ldots, P\delta y, \delta_y).$$

It can be readily seen that

$$\langle a(x), b(y)\rangle = \chi_n + \chi_{n-2} + \chi_{n-4} + \cdots .$$

Moreover, $a(x)$ and $b(x)$ are bounded by $\sqrt{n+1}$. Hence the completely bounded multiplier norm of the sum is less than or equal to $n+1$. By the triangle inequality the norm of χ_n is less than or equal to $(n+1)+(n-1)$, which gives the same estimate I got by a much more complicated method, particularly for $n > 1$.

This idea of Tadek was a starting point of our long collaboration which resulted in publishing the paper [18]. We were working on this paper in the Fall of 1982 and Spring 1983. Gradually we got an idea of using the operator P for construction of an analytic series of uniformly bounded representations of free groups. We would meet at Tadek's place mostly, and sometimes in my place to pursue the project. Tadek's wife Teresa, a maths teacher, and his sons Piotr and Tomek were at school in the mornings so we could work until early afternoon. When they returned from school we used to have dinner together. Teresa and Tadek kept open house so I could meet their friends, when I stayed longer until the evening, which occurred pretty often. At these times PCs were not available. We typed the paper on Tadek's typewriter. This was a small and fancy device, as it was possible to get a half space by holding the space key down and typing a character. In this way the machine could produce blackboard bold characters $\mathbb{R}$, $\mathbb{C}$ by combining "I" with "R" or "C". I used to get very tired in the afternoon, but Tadek worked until late at night. He was the engine of our team. Most ideas came from him. When the main part of the paper was finished we tried hard to write a proper introduction to it. Every attempt was a failure. Eventually we asked Andrzej Hulanicki who was generous enough to do the job. This is probably the best introduction I have ever had and will have in my papers.

The representations we constructed were automatically irreducible in the case of infinitely many generators. After we finished the paper and submitted it for publication, I got an idea how to decompose representations in the case of finitely many generators and was able to make the representations act on a common Hilbert space so that they depended analytically on a complex parameter. Tadek helped me in this project as an advisor, and eventually this work became my PhD thesis under Tadek's supervision. At the same time Tadek supervised the PhD thesis of a colleague of mine, Krzysztof Stempak, who is now a full professor at the Technical University of Wrocław, and is a recognized expert in harmonic analysis.

Tadek kept being popular with his students due to his crystal clear and inspiring lectures. His classes were always very well organized. I do not remember him being ever unprepared. I had the luck to hold calculus classes in 1983 when Tadek was lecturing. After one year the students of our institute organized a poll for the best lecturer and best teaching assistant. Tadek took the first position as lecturer. As a side effect of his popularity, I came first as teaching assistant. Among the students of that course, let me mention Waldemar Hebisch, Adam Sikora, Jerzy Marcinkowski, Piotr Mikrut, and Grzegorz Stachowiak. In 1997 Tadek obtained a special award for excellent teaching (see his short CV at the end of this text).

After our joint paper [18] we both worked for some time in the same area of research. There is another paper of Tadek, joint with Massimo Picardello, which I admire greatly. In [20] they gave a short and elementary proof of the famous formula of Akemann and Ostrand for the norm of the convolution operator with a function supported on a free subset in a discrete group G.

I have coauthored with Tadek another paper [23], where we showed equivalence of the series of uniformly bounded representations of the free group defined as acting on the boundary of the group and the series obtained in my PhD thesis by decomposition of representations defined on the tree as in [18].

In the late eighties I have switched to orthogonal polynomials, so our scientific contacts with Tadek gradually weakened. At the same time Tadek started a fruitful collaboration with José Galé from Saragossa, who soon became his close friend. They wrote two papers [24, 25]. In the first, they constructed a functional calculus for an infinitesimal generator of a semigroup of bounded operators analytic in the right half-space, under polynomial growth conditions on vertical lines. The novelty lied in using the Weyl fractional integration formula. The results were tested on the Poisson and Gauss semigroups in $L^1(G)$ for stratified Lie groups G. In 1995 E. B. Davies constructed a different functional calculus based on suitable bounds of the resolvent of the generator. In the follow-up paper [25] Tadek and J. Galé, together with P. J. Miana, showed that their calculus coincides with the one constructed by Davies, provided the common assumptions are satisfied. Also important applications to multiplier theory on Lie groups were given there.

Moreover, Tadek wrote several popular articles in Polish, which came out from his talks presented at various meetings in Poland. He also started writing a textbook on functional analysis for the Polish Scientific Publishers, when he was already seriously ill. The first draft was finished. He made it available to colleagues for lecturing, but he didn't manage to bring the book to its final form.

Short CV

Born:	September 30, 1944 in Katowice
Died:	May 25, 2006 in Wrocław
MSc:	1967
PhD:	1972, A construction of nuclear spaces on locally compact groups, advisor A. Hulanicki
Habilitation:	1981, Some problems of harmonic analysis on noncommutative groups
Professorship:	1991
Employment:	Mathematical Institute, University of Wrocław, since 1967
Committees:	Member of editorial committee of Colloquium Mathematicum, since 1991
PhD students:	K. Stempak, R. Szwarc
Awards:	The Stanisław Zaremba Prize of the Polish Mathematical Society, 1982
	Ministry of Science and Higher Education Prize, 1987
	National Committee for Education Medal, 1997

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[22] *Harmonic functions and Hardy spaces on trees with boundaries*, Colloq. Math. 63 (1992), 263–272.
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