Erratum

Volume 47, Number 3 (1982), in the article "Multipliers of Schatten Classes" by R. Khalil, pp. 305–313:

Let $X$ and $Y$ be given Banach spaces, and $X \otimes Y$ be the algebraic tensor product of $X$ and $Y$. For $F = \sum_{n=1}^{r} u_n \otimes v_n \in X \otimes Y$, set

$$\|F\|_p = \inf \left\{ \left( \sum_{n=1}^{r} \|u_n\|^p \cdot \|v_n\|^p \right)^{1/p} \right\},$$

where the infimum is taken over all representations of $F$ in $X \otimes Y$. In [4] Peetre and Sparr claimed that this is a norm on $X \otimes Y$, and if $X = Y = l^2$, then the completion $l^2 \otimes \rho l^2$ under the above norm is the Schatten classes $C_p$. Later on, Bergh and Lofstrom [3, p. 182] copied the claim of Peetre and Sparr. Khalil gave a (false) proof for the claim in Lemma 1.1 of [2]. Pietsch, Merdas, and Szware have pointed out that the claim of Peetre and Sparr is false by showing that for $p > 1$, $\|F\|_p = 0$ for all $F \in X \otimes Y$ even in the case when $X$ and $Y$ are one dimensional. This is because

$$\|u \otimes v\|_p \leq \left( \sum_{i=1}^{n} \frac{1}{n^{p}} \cdot \|u\|^p \cdot \|v\|^p \right)^{1/p} = n^{(1/p) - 1} \cdot \|u\| \cdot \|v\| \to 0, \text{ if } n \to \infty.$$

The use of the false claim of Peetre and Sparr does not change the results on Schur multipliers in [2].

Theorem 2.5 in [2] (which is the only theorem where the claimed representation of $C_p$ is used) is now restated and reproved:

**Theorem A.** Let $\varphi$ be an infinite matrix in $M(C_p)$ and $\psi$ be another matrix obtained from $\varphi$ by repeating the first column. Then

$$| \langle \varphi \cdot \varphi, u \otimes v \rangle | \leq \|\varphi\|_M \cdot \|\psi\|_p \cdot \|u \otimes v\|_p^*,$$

where $\| \cdot \|_p^*$ is the usual norm on $C_p$ and $u, v \in l^2$.

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Proof. For $u \in l^2$, set $u(1) = (|u(0)|^2 + |u(1)|^2)^{1/2}$ and $\tilde{u}(i) = u(i)$ for $i \geq 2$. Assume $\|u\|_2 \leq 1$. Set $Q$ to denote the matrix

$$
Q = \begin{pmatrix}
\sin \alpha & 0 \\
\cos \alpha & 0 \\
0 & 1
\end{pmatrix},
$$

where $\cos \alpha = u(0)/\tilde{u}(1)$, $\sin \alpha = u(1)/\tilde{u}(1)$.

By Lemma 2.2 in [2], then $|\langle \varphi \cdot \psi, u \otimes v \rangle| = |\langle \varphi \cdot (\psi \circ Q), \tilde{u} \otimes v \rangle| \leq \|\varphi \cdot (\psi \circ Q)\|_\infty \cdot \|\tilde{u} \otimes v\|_1 \leq \|\varphi\|_m \cdot \|\psi \circ Q\|_p \cdot \|\tilde{u}\|_2 \cdot \|v\|_2$. Since $\|Q\| \leq 1$, we get

$$
|\langle \varphi \cdot \psi, u \otimes v \rangle| \leq \|\varphi\|_m \cdot \|\psi\|_p \cdot \|u \otimes v\|_{p^*}.
$$

Q.E.D.

Now, in Theorem 2.6 of [2], Theorem 2.5 was used. But only the new version stated in this note was used, more precisely [2, p. 311, lines 4 and 7]:

$$
\|\tilde{\varphi} \cdot \vartheta \otimes w_1\|_{p^*} \leq \|\varphi\|_M.
$$

This now goes as follows:

$$
|\langle \tilde{\varphi} \cdot \vartheta \otimes w_1, \psi \rangle| = |\langle \tilde{\varphi} \cdot \psi, \vartheta \otimes w_1 \rangle| \leq \|\varphi\|_M \cdot \|\psi\|_p \cdot \|\vartheta \otimes w_1\|_{p^*}
$$

(by Theorem A). Since $\|\vartheta\| = \|w_1\| = 1$, and $\psi$ is any element in $C_p$, it follows that $\|\tilde{\varphi} \cdot \vartheta \otimes w_1\|_{p^*} \leq \|\varphi\|_M$.

Final Remark. The statement of Lemma 1.3 in [2] is true. A proof is given in [6, Corollary 4.3, p. 44].

REFERENCES


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