JOURNAL OF FUNCTIONAL ANALYSIS 63, 137–138 (1985)

Erratum

Volume 47, Number 3 (1982), in the article "Multipliers of Schatten Classes" by R. Khalil,* pp. 305–313:

Let X and Y be given Banach spaces, and $X \otimes Y$ be the algebraic tensor product of X and Y. For $F = \sum_{n=1}^{r} u_n \otimes v_n \in X \otimes Y$, set

$$\|F\|_p = \inf\left\{\left(\sum_{n=1}^{r} \|u_n\|^p \cdot \|v_n\|^p\right)^{1/p}\right\},$$

where the infimum is taken over all representations of F in $X \otimes Y$. In [4] Peetre and Sparr claimed that this is a norm on $X \otimes Y$, and if $X = Y = l^2$, then the completion $l^2 \hat{\otimes}_p l^2$ under the above norm is the Schatten classes C_p. Later on, Bergh and Löfström [3, p. 182] copied the claim of Peetre and Sparr. Khalil gave a (false) proof for the claim in Lemma 1.1 of [2]. Pietsch, Merdas, and Szware have pointed out that the claim of Peetre and Sparr is false by showing that for $p > 1$, $\|F\|_p = 0$ for all $F \in X \otimes Y$ even in the case when X and Y are one dimensional. This is because

$$\|u \otimes v\|_p \leqslant \left(\sum_{i=1}^{n} \frac{1}{n^p} \cdot \|u\|^p \cdot \|v\|^p\right)^{1/p} = n^{(1/p)-1} \cdot \|u\| \cdot \|v\| \to 0, \text{ if } n \to \infty.$$

The use of the false claim of Peetre and Sparr does not change the results on Schur multipliers in [2].

Theorem 2.5 in [2] (which is the only theorem where the claimed representation of C_p is used) is now restated and reproved:

THEOREM A. *Let φ be an infinite matrix in $M(C_p)$ and φ be another matrix obtained from φ by repeating the first column. Then*

$$|\langle \tilde{\varphi} \cdot \varphi, u \otimes v \rangle| \leqslant \|\varphi\|_M \cdot \|\psi\|_p \cdot \|u \otimes v\|_{p^*},$$

where $\| \ \|_p$ is the usual norm on C_p and $u, v \in l^2$.

* The authors would like to thank Professor L. Gross for an excellent job of editing.

0022-1236/85 $3.00

Proof. For $u \in l^2$, set $u(1) = (|u(0)|^2 + |u(1)|^2)^{1/2}$ and $\tilde{u}(i) = u(i)$ for $i \geqslant 2$. Assume $\|u\|_2 \leqslant 1$. Set Q to denote the matrix

$$Q = \left(\begin{array}{c|c} \begin{smallmatrix} \sin\alpha \\ \cos\alpha \end{smallmatrix} & 0 \\ \hline 0 & 1 \end{array}\right),$$

where $\cos\alpha = u(0)/\tilde{u}(1)$, $\sin\alpha = u(1)/\tilde{u}(1)$.

By Lemma 2.2 in [2], then $|\langle \tilde{\varphi} \cdot \psi,\ u \otimes v \rangle| = |\langle \varphi \cdot (\psi \circ Q),\ \tilde{u} \otimes v \rangle| \leqslant \|\varphi \cdot (\psi \otimes Q)\|_\infty \cdot \|\tilde{u} \otimes v\|_1 \leqslant \|\varphi\|_m \cdot \|\psi \circ Q\|_p \|\tilde{u}\|_2 \|v\|_2$. Since $\|Q\| \leqslant 1$, we get $|\langle \tilde{\varphi} \cdot \psi,\ u \otimes v \rangle| \leqslant \|\varphi\|_m \cdot \|\psi\|_p \cdot \|u \otimes v\|_{p^*}$. Q.E.D.

Now, in Theorem 2.6 of [2], Theorem 2.5 was used. But *only the new version* stated in this note was used, more precisely [2, p. 311, lines 4 and 7]:

$$\|\tilde{\varphi} \cdot \theta \otimes w_1\|_{p^*} \leqslant \|\varphi\|_M.$$

This now goes as follows:

$$|\langle \tilde{\varphi} \cdot \theta \otimes w_1, \psi \rangle| = |\langle \tilde{\varphi} \cdot \psi, \theta \otimes w_1 \rangle| \leqslant \|\varphi\|_M \cdot \|\psi\|_p \cdot \|\theta \otimes w_1\|_{p^*}$$

(by Theorem A). Since $\|\theta\| = \|w_1\| = 1$, and ψ is any element in C_p, it follows that $\|\tilde{\varphi} \cdot \theta \otimes w_1\|_{p^*} \leqslant \|\varphi\|_M$.

Final Remark. The statement of Lemma 1.3 in [2] is true. A proof is given in [6, Corollary 4.3, p. 44].

References

1. I. C. Gohberg and M. G. Krein, Introduction to the theory of linear nonself-adjoint operators, *Trans. Math. Monographs* **18** (1969).
2. R. Khalil, Multipliers of Schatten classes, *J. Funct. Anal.* **47** (1982), 305–313.
3. J. Bergh and J. Löström, "Interpolation Spaces," Springer-Verlag, New York, 1976.
4. J. Peetre and G. Sparr, Interpolation of normed abelian groups, *Ann. Mat. Pura Appl.* **92** (1972), 217–262.
5. A. Pietsch, Grothendieck's concept of a p-nuclear operator, preprint.
6. Q. Stout, Ph. D. thesis, Indiana Univ., 1976.

R. Khalil
B. Merdas
R. Szware

Printed in Belgium