Let s_n be indeterminate moment sequence and let μ be a solution of the moment problem. The inequality

$$\sum_{n,m=0}^{N} s_{n+m} a_n \overline{a_m} \geqslant c \sum_{k=0}^{N} |a_k|^2$$

is equivalent to

$$\int \left| \sum_{k=0}^{N} a_k x^k \right|^2 \, d\mu(x) \geqslant c \sum_{k=0}^{N} |a_k|^2.$$

Let

$$\sum_{k=0}^{N} a_k x^k = \sum_{n=0}^{N} c_n P_n(x)$$

and

$$P_n(x) = \sum_{k=0}^{n} b_{k,n} x^k. \tag{1}$$

Then

$$\sum_{n=0}^{N} |c_n|^2 \geqslant c \sum_{k=0}^{N} \left| \sum_{n=k}^{N} b_{k,n} c_n \right|^2$$

Therefore $c > 0$ is equivalent to the fact that the upper triangular matrix

$$B = (b_{k,n}), \qquad b_{k,n} = 0, \; k > n.$$

corresponds to a bounded operator on ℓ^2. From (1) we have

$$b_{k,n} = \frac{1}{2\pi i} \int_{|z|=r} P_n(z) z^{-(k+1)} \, dz = r^{-k} \frac{1}{2\pi} \int_0^{2\pi} P_n(re^{it}) e^{-ikt} \, dt. \tag{2}$$

Consider $r = 1$. Then by Parseval identity we have

$$\sum_{n=0}^{\infty} \sum_{k=0}^{n} |b_{k,n}|^2 = \sum_{n=0}^{\infty} \frac{1}{2\pi} \int_0^{2\pi} |P_n(e^{it})|^2 \, dt$$

Therefore the operator B is Hilbert-Schmidt. Hence both B^*B and BB^* are of trace class.

It is possible to show much stronger property of B. For example B is of trace class. Indeed, by (2) and by Parseval identity we have

$$\sum_{n=0}^{\infty} \sum_{k=0}^{n} r^{2k} |b_{k,n}|^2 = \sum_{n=0}^{\infty} \frac{1}{2\pi} \int_0^{2\pi} |P_n(re^{it})|^2 \, dt < \infty$$

For $r > 1$ we obtain

$$\left(\sum_{k=0}^{n} |b_{k,n}|\right)^2 \leqslant \sum_{k=0}^{n} r^{-2k} \sum_{k=0}^{n} r^{2k}|b_{k,n}|^2 \leqslant \frac{r^2}{r^2-1} \sum_{k=0}^{n} r^{2k}|b_{k,n}|^2$$

Thus

$$\sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} |b_{k,n}|\right)^2 \leqslant \frac{r^2}{r^2-1} \sum_{n=0}^{\infty}\sum_{k=0}^{n} r^{2k}|b_{k,n}|^2 < \infty. \tag{3}$$

Moreover

$$\sum_{n=0}^{\infty}\sum_{k=0}^{n} r^{2k}|b_{k,n}|^2 = \sum_{k=0}^{\infty} r^{2k} \sum_{n=k}^{\infty} |b_{k,n}|^2.$$

Hence

$$\sum_{k=0}^{\infty} \sqrt{\sum_{n=k}^{\infty} |b_{k,n}|^2} \leqslant \frac{r}{\sqrt{r^2-1}} \sqrt{\sum_{k=0}^{\infty} r^{2k} \sum_{n=k}^{\infty} |b_{k,n}|^2} < \infty, \tag{4}$$

which implies that B^* is of trace class. This is because denoting by δ_k the standard basis in ℓ^2 gives that

$$\sum_{k=0}^{\infty} \|B^*\delta_k\|_2 < \infty.$$

Hence also B is of trace class.

Inequality (3) implies that B^* maps continuously ℓ^∞ into ℓ^2 while (4) gives that B maps continuously ℓ^2 into ℓ^1. The latter follows also by duality from (3). In this way BB^* is a bounded map from ℓ^∞ into ℓ^1. This property is much stronger than the trace class.