Obligatory Exercises

Exercise 1. Generate n observations from the distribution of $N(\mu, \sigma^2)$.

(a) $n = 50, \quad \theta = 1, \quad \sigma = 1,$

(b) $n = 50, \quad \theta = 4, \quad \sigma = 1,$

(c) $n = 50, \quad \theta = 1, \quad \sigma = 2.$

On this basis, calculate the value of the estimator of the parameter θ of the form

(i) $\hat{\theta}_1 = \bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$

(ii) $\hat{\theta}_2 = Me\{X_1, \ldots, X_n\},$

(iii) $\hat{\theta}_3 = \sum_{i=1}^{n} w_i X_i, \quad$ where $\sum_{i=1}^{n} w_i = 1, \ 0 \leqslant w_i \leqslant 1, \ i = 1, \ldots, n,$ with your own choice of weights.

(iv) $\hat{\theta}_4 = \sum_{i=1}^{n} w_i X_{i:n}, \quad$ where $X_{1:n} \leqslant \ldots \leqslant X_{n:n}$ are ordered observations $X_1, \ldots, X_n,$

$$w_i = \varphi\left(\Phi^{-1}(\frac{i-1}{n})\right) - \varphi\left(\Phi^{-1}(\frac{i}{n})\right),$$

where φ is the density of the the standard normal distribution $N(0,1)$ and Φ is the cumulative distribution function.

Repeat experience 10,000 times. On this basis, estimate the variance, mean square error and the load on each of the estimators. Discuss the results obtained.

Exercise 2. Generate n observations from the logistic distribution $L(\theta, \sigma)$ with the shift parameter θ and the scale parameter σ.

(a) $n = 50, \quad \theta = 1, \quad \sigma = 1,$

(b) $n = 50, \quad \theta = 4, \quad \sigma = 1,$

(c) $n = 50, \quad \theta = 1, \quad \sigma = 2.$

Estimate the value of the maximum likelihood estimator of the parameter θ based on the generated sample. Discuss the choice of the starting point and the number of steps in the algorithm. Repeat experience 10,000 times. On this basis, estimate the variance, mean square error and the bias on the estimator. Discuss the obtained results.

Exercise 3. Generate n observations from the Cauchy distribution $C(\theta, \sigma)$ with the shift parameter θ and the scale parameter σ.

(a) $n = 50, \quad \theta = 1, \quad \sigma = 1,$

(b) $n = 50, \quad \theta = 4, \quad \sigma = 1,$

(c) $n = 50, \quad \theta = 1, \quad \sigma = 2.$

Estimate the value of the maximum likelihood of the parameter θ based on the generated sample. Discuss the choice or the starting point and the number of steps in the algorithm. Repeat experience 10,000 times. On this basis, estimate the variance, mean square error and the load on the estimator. Discuss the obtained results.

ADDITIONAL EXCERCISES

Exercise 4. Discuss the necessity of numerical determination of the estimators of the highest reliability, for example estimation of the shift parameter in the logistic distribution (example 6.1.4, p. 315, Hogg et al., 2005).

Exercise 5. Discuss the chosen numerical method allowing the estimation of the maximum likelychood estimator dignity (see, e.g., p. 329, Hogg et al., 2005).

Exercise 6. Repeat the numerical experiment from the tasks 1, 5 and 6 for $n = 20$ and $n = 100$. Discuss the obtained results in relation to previous results.

Literature.

Hogg, R. V., McKean, J. W., Craig, A. T. (2005). *Introduction to Mathematical Statistics.* Pearson Education International, London.

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