

Zadania i problemy do wykładu Matematyka dla specjalności GiBE
(ZESTAW NR 1)

ZADANIA

Zadanie 1. Wstaw znak równości lub odpowiedniej nierówności:

(a) $3^{\frac{2}{3}} \bigcirc 3^{\frac{3}{4}}$	(b) $\left(\frac{1}{4}\right)^{\frac{1}{3}} \bigcirc \left(\frac{1}{2}\right)^{\frac{1}{6}}$
(c) $(2^3 - 2^2) \bigcirc (3^2 - 2^2)$	(d) $\left(\frac{1}{3}\right)^{34} \bigcirc \left(\frac{1}{9}\right)^{17}$

Zadanie 2. Oblicz:

(a) $18^{\frac{1}{2}} \cdot 2^{\frac{1}{2}}$	(b) $3^{\frac{1}{2}} \cdot 27^{\frac{1}{2}}$	(c) $9^{\frac{1}{2}} \cdot 27^{-\frac{1}{6}}$
(d) $7^{\frac{1}{2}} \cdot 7^{\frac{1}{3}} \cdot 7^{\frac{1}{6}}$	(e) $\left(5^{\frac{3}{2}} \cdot \sqrt[3]{5}\right)^4$	(f) $\left(3^3 \cdot \sqrt[6]{9}\right)^{\frac{3}{4}}$
(g) $\frac{2^{-3} \cdot 4^6}{2^7}$	(h) $\frac{5^5 \cdot \left(\frac{1}{5}\right)^4}{5^{-7}}$	(i) $\frac{11^{\frac{3}{2}}}{\sqrt{44}} \cdot \left(\frac{2}{11}\right)^3$

Zadanie 3. Sprowadź do najprostrzej postaci:

(a) $(\sqrt[3]{x} - \sqrt[3]{y}) \left(x^{\frac{2}{3}} + \sqrt[3]{xy} + y^{\frac{2}{3}}\right)$	(b) $\left(x^{-\frac{1}{2}} + y^{-\frac{1}{2}}\right) \left(\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{y}}\right)$
(c) $\left(a^{\frac{1}{3}} \cdot b^{\frac{1}{6}}\right)^{-2} \left(a^{-\frac{1}{3}} \cdot b^{-\frac{2}{3}}\right)^3$	(d) $\frac{\sqrt[3]{x} \sqrt[5]{y}}{x^{\frac{5}{3}} y^{\frac{3}{5}}}$

Zadanie 4. Która z liczb jest większa:

(a) $x = \left(3^{-3}\right)^{-2}, \quad y = (2\sqrt{5} + 2)^{\frac{1}{2}}(2\sqrt{5} - 2)^{\frac{1}{2}}$	(b) $x = 7^{1-\pi} \cdot \frac{1}{7^\pi}, \quad y = 7^{-6}$
(d) $x = 3^{\frac{1}{2}} \cdot 12^{-\frac{1}{2}}, \quad y = 9^{-\frac{3}{4}} \cdot 9^{\frac{1}{4}} \cdot 81^{\frac{3}{4}}$	(c) $x = \left(5^{\sqrt{2}}\right)^{\sqrt{2}}, \quad y = 125^{\frac{2}{3}}$

Zadanie 5. Oblicz:

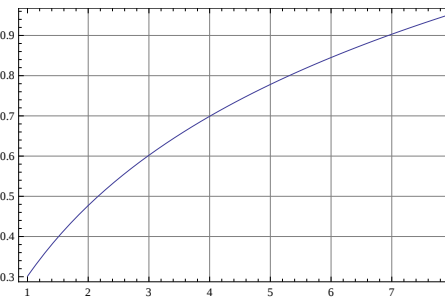
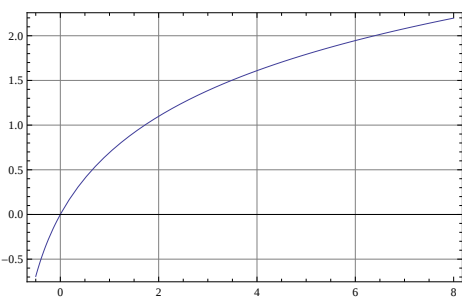
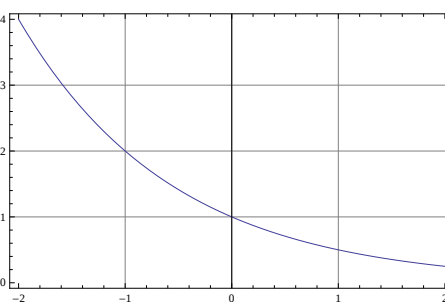
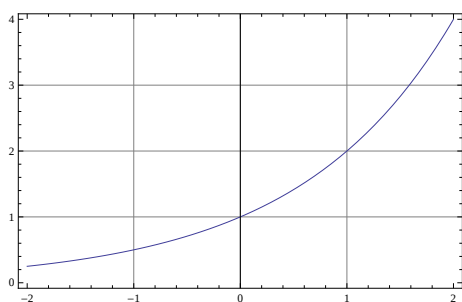
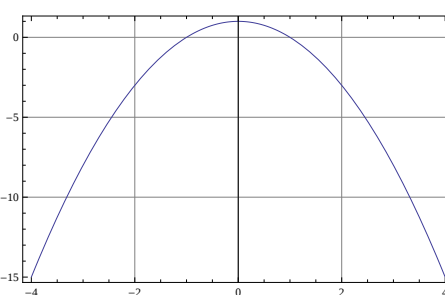
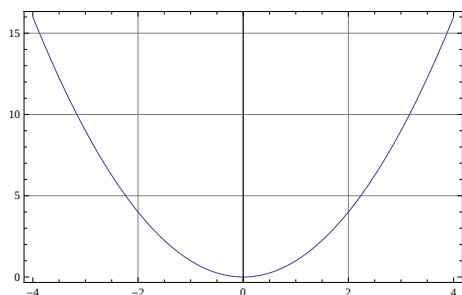
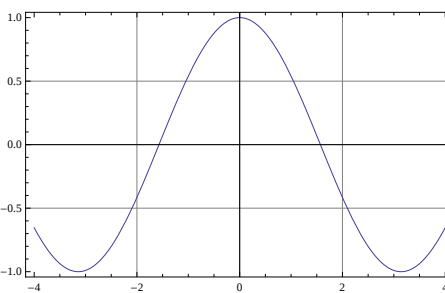
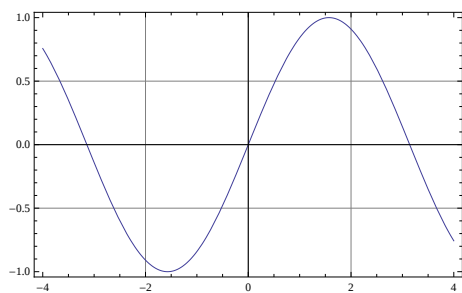
(a) $\log_2 32$	(b) $\log_2 \frac{1}{8}$	(c) $\left(\frac{1}{9}\right)^{\log_3 \sqrt{5}}$
(d) $\log_3 27$	(e) $\log_4 \frac{1}{\sqrt{2}}$	(f) $125^{\log_5 \sqrt[3]{2}}$
(g) $2^{\log_2 7}$	(h) $\log_3 \frac{\sqrt{3}}{9}$	(i) $\log_3 \sqrt[3]{3} - \log_7 \sqrt[3]{7}$

Zadanie 6. Jaki wykres w skali logarytmicznej będą miały funkcje:

(a) $f(x) = 2^x$	(b) $f(x) = x^3$	(c) $f(x) = 2014^{x^2}$	(d) $f(x) = \sqrt[3]{x}$
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Zadanie 7. Który rysunek przedstawia wykres funkcji:

(i) $\sin(x)$, (ii) x^2 , (iii) $\cos(x)$, (iv) $1 - x^2$, (v) $\ln(1 + x)$, (vi) 2^x , (vii) 2^{-x} .



Zadanie 8. Oblicz:

(a) $\ln\left(\frac{x}{y}\right) + \ln\left(\frac{y}{x}\right)$

(b) $\ln(xyz) - \ln(y^2z) + \ln\left(\frac{y}{x}\right)$

(c) $3^{\log_3 \sqrt{3} 27}$

(d) $\log_3 \frac{1}{12} + \log_3 \frac{14}{15} + \log_3 \frac{10}{21}$