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A Markov Chain Model of Day to Day Changes in the Canadian Forest Fire Weather Index

David L. Martell

Faculty of Forestry, University of Toronto

33 Willcocks Street, Toronto, Ontario, Canada, M5S 3B3

Phone: +1 416 978 6960; Fax: +1 416 978 3834; email: martell@smokey.forestry.utoronto.ca

Abstract

A Markov chain is used to model day to day changes in the Fire Weather Index (FWI) component of the Canadian Forest Fire Weather Index System. The results of statistical analyses of 26 years (1963 through 1988) of fire weather data recorded at 15 fire weather stations located across the province of Ontario suggest that it is reasonable to partition the fire season into three subseasons and model day to day changes in the Fire Weather Index class within each subseason as a Markov chain of order 1.

Keywords:

Markov process

Fire weather

Forest fire management.

Introduction

Weather has a very significant impact on the occurrence, behaviour, and impact of fires on forest ecosystems. Fire danger rating systems like the Canadian Forest Fire Weather Index System (CFFWIS) transform daily weather observations into relatively simple indices that can be used to predict fire occurrence, behaviour, and impact (Stocks et al. 1989). They are used for many purposes including planning for the daily deployment of fire suppression resources and the evaluation of fire management strategies. They can also be incorporated in ecosystem models to assess the long-term implications of specified fire management policies and fire regimes.

Since forest fires have the potential to burn for many days, fire management system models must address the important fact that what happens during a particular day can be influenced significantly by what has taken place on the preceding days. Consider, for example, deployment models that are used to evaluate fire suppression resource strategies over 2–3-day planning horizons. Each day a small proportion of the fires that are reported may escape initial attack and become extended attack fires that demand large amounts of suppression resources for several days. The fraction of fires that escape each day will depend upon many factors including the fire danger rating indices that are based upon current and past weather, and the number and quality of initial attack crews available that day. Fire managers often refer to the occurrence of several consecutive days of elevated fire danger during which many fires happen as a 'fire flap'. The probability that a fire will escape will usually increase as the duration of the fire flap increases, fire crews and other suppression resources become less readily available, and crews that are available become fatigued. Fire management planning models that treat days independently will not capture the build-up and subsequent increased fire escape probability associated with fire flaps, and their use could lead to under-estimates of fire suppression resource needs and area burned.

The ability to model day to day changes in fire weather is also emerging as an important issue as more effort is devoted to the development of models designed to investigate the impact of fire and fire management regimes on forest ecosystems at the landscape level. Consider, for example, forest simulation models with fire growth models that are used to predict the spatial and temporal impacts of fire on boreal forest ecosystems. Since topography and vegetation do not vary throughout the potential life of a fire, the validity of such models depends in part upon the accuracy with which day to day changes in fire weather are modelled.

Although they should account for the fact that the current fire danger depends in part on what has transpired in the past, fire management planners do not always do so. Markov chains are mathematical models that can be used to model sequential dependencies that influence the behaviour of probabilistic dynamic systems, and the mathematical properties of Markovian models can be exploited to develop relatively simple tractable models of such systems. They also make possible the development of Markov decision process models that can be used to develop strategic planning models that can be used to evaluate policies for managing Markovian systems. If Markov models can be used to model day to day changes in fire danger rating indices, the properties of Markov chains and the rich Markov decision process literature can be exploited to develop fire management plan-

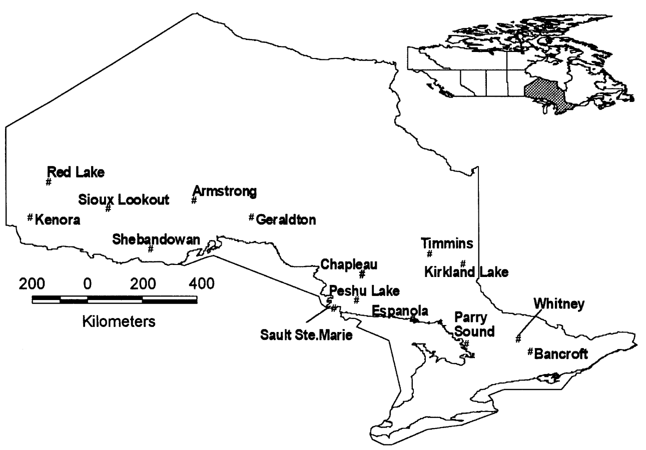

Figure 1. The province of Ontario, showing the 15 weather stations where the data were collected.

ning systems that account for the day to day dependence that is so important when dealing with fire danger.*

Meteorologists have used Markov chains to model sequences of wet and dry days (see for example, Gabriel and Neumann (1962), Gates and Tong (1976), and Stern and Coe (1984)). Markov chains have also been used to model day to day changes in fire danger rating indices for fire management planning purposes. Kourtz (1970) used historical weather data for a forested area in Idaho and Montana to model thunderstorm day occurrence as a first order Markov chain, and day to day changes in a spread index class variable as a second order Markov chain, but he did not report the results of statistical tests of the validity of either model. Martell (1971) used a first-order Markov chain to model day to day changes in the average burning index across Ontario's Northwestern Region but did not carry out statistical tests of validity of his model. Greulich (1976) used likelihood ratio statistical test results to justify his use of a first order Markov chain to model day to day changes in the brush burning index class at two fire weather stations located in the California Division of Forestry's District 1. Boychuk and Martell (1988) assumed that day to day changes in a fireload index based in part, on the FWI, could be modelled as a Markov chain, and incorporated that model in a larger Markov model that was used to evaluate annual fire fighter hiring decisions.

Despite the potential value of and demonstrated interest in Markovian fire weather models, only Greulich (1976) who analysed 9 years of data observed at two weather stations in California and Martell (1989), who analysed 26 years of data observed at three weather stations in Ontario, have presented the results of formal statistical analyses of their models. Those small scale studies support the use of Markov chain models of fire danger rating indices but much closer scrutiny is required to justify the ongoing use of the Markovian assumption to develop fire management planning models. The purpose of this paper is to present the results of a comprehensive statistical analysis of the validity of Markov fire danger rating system models based on 26 year sequences of historical fire weather data observed at the 15 widely separated Ontario fire weather stations depicted in Figure 1.

Markov Chains and Fire Weather

Fire danger rating indices are designed to track one or more important aspects of the fire environment such as forest fuel moisture, with functions that relate the current day's indices to the previous day's indices and selected weather variables such as the total rainfall observed during the intervening time period. They are usually measured on continuous scales but they are often classified into discrete categories (e.g. nil, low, moderate, high, or extreme fire danger rating classes) for planning purposes. It is therefore reasonable to model day to day changes in the fire danger rating index class as a Markov chain.†

Suppose each day of a fire season can be classified with respect to some fire danger rating index, and $X_n=i$ indicates that the system is in class i on day n. A Markov chain of order 0 is one for which tomorrow's state is independent of today's state and all previous states. A Markov chain of order 1 has the property that the conditional probability distribution of X_{n+1} (i.e. the state of the system on day n+1) given X_1, $X_2,\ldots,X_n$ depends only on X_n. In simple terms, the probability that the Fire Weather Index on a particular day will be in a specified class depends only on its class the previous day, and the earlier days can be ignored. This Markov property can be expressed mathematically as

$$P\{X_{n+1}=j|\, X_1=i_1,\, X_2=i_2,\ldots,X_n=i\} = P\{X_{n+1}=j|\, X_n=i\}$$

for $n = 1,2,\ldots$ and every sequence $i_1,i_2,\ldots,i,j$.

A Markov chain of order 1 with m states can be characterized by its state transition probability matrix $\boldsymbol{P}$, and P_{ij}, the element in the ith row and jth column of the $m \times m$ matrix $\boldsymbol{P}$ is the probability that the system (e.g. the fire danger rating class) will be in state j tomorrow, given that it is in state i today. A Markov chain of order 2 is one for which the conditional probability distribution of the state of the system tomorrow depends on the state of the system today and yesterday. A Markov chain of order r is one for which the conditional probability distribution of the state of the system tomorrow depends on the state of the system during the past r days.

* Markov decision process models constitute a very rich and powerful body of knowledge that can be applied to fire management planning and the development of fire impact assessment models. Most operations research texts such as Winston (1994) contain brief introductions to Markov decision processes. Puterman (1994) presents a comprehensive treatment of the subject and describes applications in ecology, economics and other areas.

† For an introduction to Markov chain theory see for example, Bhat (1984) or Ross (1989).

A Markov Chain Model of the Canadian Forest Fire Weather Index

Canadian forest fire management agencies use the Canadian Forest Fire Weather Index System (CFFWIS) for fire danger rating purposes. The Fire Weather Index (FWI) is one of six components of the CFFWIS which is described in detail by Van Wagner (1987). The CFFWIS has three basic moisture codes that are numerical ratings of the moisture content of three different components of a model forest fuel complex: the Fine Fuel Moisture Code (FFMC), the Duff Moisture Code (DMC), and the Drought Code (DC). Each day's FFMC is based on the previous day's FFMC and noon observations of temperature, relative humidity, wind speed, and the 24 h rainfall. The FFMC thus provides a running 'inventory' of the moisture content of the fine fuels and is designed such that the current day's FFMC is closely related to the previous day's FFMC. The DMC and DC are designed to reflect the moisture content of other components of the model fuel complex and are partially based on the previous day's DMC and DC respectively. It is therefore reasonable to assume day to day changes in the FFMC, DMC and DC can be modelled as Markov processes.

The Initial Spread Index (ISI), which is designed to be a numerical rating of the spread rate of a fire, is a function of current day's FFMC and the noon hour wind speed. The Buildup Index (BUI) is a relative measure of the amount of fuel available for combustion and is a function of the current day's DMC and DC. They in turn, are used to compute the FWI each day. The Ontario Ministry of Natural Resources (OMNR) uses the classification scheme described in Table 1 to classify each day's FWI as nil, low, moderate, high, or extreme. These relationships and the fact that daily precipitation can be modelled as a Markov process suggest it may be reasonable to model day to day changes in the FWI class as a Markov chain.

Statistical Tests for a Markov Chain Model for the FWI Class

In order to model day to day changes in the FWI class it is necessary to decide upon the order of the Markov chain and estimate the state transition probabilities. Since the FWI is based on weather and, to a minor extent, the calendar date, it is reasonable to assume that the elements of the probability matrix $\boldsymbol{P}$ vary over the course of the season. The fire season was therefore partitioned into a small number of subseasons within which it is reasonable to assume that $\boldsymbol{P}$ is homogeneous with respect to time. Although this approach does not capture all the non-homogeneous behaviour of the FWI, it is analogous to using a step function to represent a continuous function and simplifies the statistical analysis of what is assumed to be a homogeneous Markov chain within each subseason.

Table 1. Fire Weather Index classification scheme used in Ontario.

Range	FWI Class
FWI = 0	Nil
0 < FWI ≤ 4	Low
4 < FWI ≤ 11	Moderate
11 < FWI ≤ 23	High
23 < FWI	Extreme

To use this approach it was necessary to subjectively identify periods within which $\boldsymbol{P}$ appeared to be relatively homogeneous. Twenty-six years of historical fire weather data from 15 OMNR fire weather stations were used to determine an average FWI for each weather station for each week of the fire season beginning on 15 April (the first day of week 1) and ending on 6 October (the last day of week 25). The average FWI for a specific weather station for a particular week (e.g. week 1; April 15 through April 21 inclusive) is the arithmetic average of all the FWI values observed at that station during that week over the 26 year period. In most cases (unless there were missing observations) there were in total 182 observations for each week. The average FWI was plotted as a function of the week and an attempt was made to subjectively delineate periods during which the average FWI increased, decreased, or remained relatively stable. Each fire weather station was analysed independently, and spatial correlation between the widely separated stations was not investigated.

Figure 2 is the graph of the weekly average FWI for the Kenora weather station, the most westerly fire weather station that was studied. Figure 3 is a similar graph of the weekly average FWI observed at Shebandowan which is located in west central Ontario. Figure 4 is a graph of the weekly average FWI for Kirkland Lake which is located in north-eastern Ontario. There were some missing observations, particularly at the beginning and the end of the fire season. Furthermore, the precise 'start' (the time at which the winter snowpack has melted and the forest vegetation has become dry enough to support combustion) and 'end' of the fire season varies from year to year and place to place in response to large scale atmospheric circulation patterns. The first two weeks (15 April 28 through April) and last three weeks (16 September through 6 October) of the fire season were therefore excluded from further analysis. The graphs in Figures 2, 3 and 4 were subjectively assessed and it appeared to be reasonable to partition the 26 April through 15 September portion of the fire season into the three subseasons described in Table 2, which were used in an earlier fire occurrence prediction study in Ontario (Martell et al. 1987).

The following procedure, which is based on the likelihood ratio tests described in Bhat (1984, pp. 136–144) was then used to determine an appropriate Markov chain order for each subseason and fire weather station. The first step was to consider the hypotheses H_O and H_A

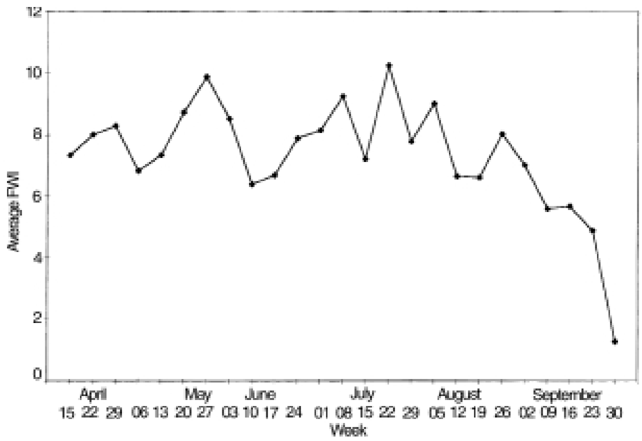

Figure 2. Average FWI in Kenora District, 1963–1988.

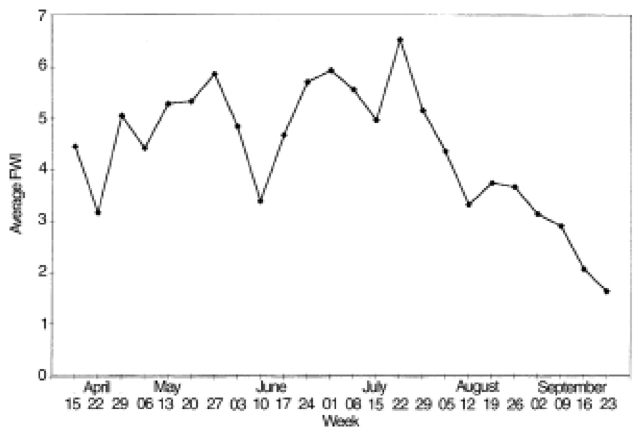

Figure 3. Average FWI in Shebandowan District, 1963–1988.

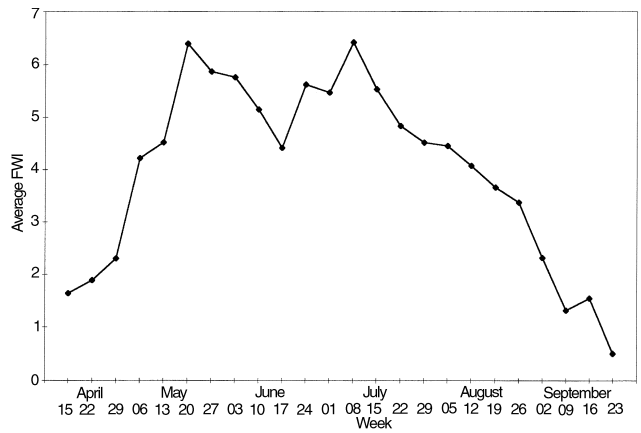

Figure 4. Average FWI in Kirkland District, 1963–1988.

H_O : Markov chain is of order 0 or $P_{ij} = P_j$

H_A : Markov chain is of order 1.

The likelihood ratio test statistic $S_{0,1}$ is

$$S_{0,1} = 2 \sum_{ij} n_{ij} [\ln \hat{P}_{ij} - \ln \hat{P}_j]$$

and has a χ^2 distribution with $(m-1)^2 - d$ degrees of freedom, where

m = number of FWI classes,

n_{ij} = number of transitions from state i to state j,

n_j = number of times state j was observed,

n = total number of observations,

Table 2. Classification scheme that was used to partition the fire season into three subseasons.

Subseason	Weeks	Dates
Spring	3–8	29 April– 9 June
Early summer	9–15	10 June–28 July
Summer	16–22	29 July–15 Sept.

$\hat{P}_{ij} = n_{ij} / \sum_j n_{ij}$

$\hat{P}_j = n_j / n$, and

d = number of $\hat{P}_j$ values that equal zero and only $\hat{P}_{ij}$ values corresponding to non-zero $\hat{P}_j$ values are used.

If $S_{0,1}$ is greater than χ^2_α with $(m-1)^2 - d$ degrees of freedom, then H_O is rejected at significance level α and the process continues. If H_O is not rejected it is reasonable to use a Markov chain of order 0 (i.e. to assume tomorrow's FWI class is independent of today's FWI class) and terminate the procedure. The test was conducted for $\alpha = 0.01$ and $\alpha = 0.05$.

The second step considered the hypotheses H_O and H_A:

H_O : Markov chain is of order 1 or $P_{ijk} = P_{jk}$

H_A : Markov chain is of order 2.

The likelihood ratio test statistic $S_{1,2}$ is

$$S_{1,2} = 2 \sum_{ijk} n_{ijk} [\ln \hat{P}_{ijk} - \ln \hat{P}_{jk}]$$

and has a χ^2 distribution with $m(m-1)^2 - d$ degrees of freedom, where

n_{ijk} = number of transitions from state i to state j and then state k,

Table 3. Results of the likelihood ratio statistical tests concerning the rank of Markov chain models for day to day changes in the FWI class, based on fire weather data for the 1963 through 1988 fire seasons.

			Likelihood ratio statistics and degrees of freedom						Suggested Markov chain order given α	
Weather station	Subseason	Sample size	$S_{0,1}$	df	$S_{1,2}$	df	$S_{2,3}$	df	α =0.05	α =0.01
Red Lake	Spring	983	629.8	16	104.1	79	215.8	364	2	1
	Early Summer	1274	565.3	16	96.3	80	310.6	375	1	1
	Summer	1274	711.0	16	86.5	79	194.6	361	1	1
Sioux Lookout	Spring	1006	680.8	16	95.8	80	237.6	368	1	1
	Early Summer	1274	669.3	16	67.2	79	266.4	371	1	1
	Summer	1273	741.7	16	73.7	78	218.3	360	1	1
Kenora	Spring	1034	643.0	16	90.0	79	268.9	370	1	1
	Early Summer	1274	673.1	16	82.2	79	265.3	379	1	1
	Summer	1250	730.5	16	119.4	79	255.3	368	2	2
Shebandowan	Spring	1056	713.2	16	86.0	78	173.3	349	1	1
	Early Summer	1274	678.6	16	80.3	79	189.5	360	1	1
	Summer	1261	725.4	16	69.6	76	156.3	343	1	1
Armstrong	Spring	985	669.9	16	97.8	79	214.8	358	1	1
	Early Summer	1274	661.5	16	104.8	78	213.1	359	1	1
	Spring	1268	778.8	16	94.2	78	200.2	352	1	1
Geraldton	Spring	963	574.6	16	82.9	79	165.5	358	1	1
	Early Summer	1274	533.8	16	87.2	78	220.1	358	1	1
	Summer	1274	627.7	16	100.1	78	173.0	351	2	1
Chapleau	Spring	960	564.3	16	80.0	79	212.2	363	1	1
	Early Summer	1257	575.7	16	97.3	79	243.0	370	1	1
	Summer	1198	701.1	16	116.7	79	218.0	358	2	2
Timmins	Spring	932	535.8	16	60.7	78	214.9	365	1	1
	Early Summer	1273	673.1	16	56.2	77	188.6	357	1	1
	Summer	1226	551.0	16	68.6	74	128.0	335	1	1
Kirkland Lake	Spring	990	646.4	16	97.6	80	171.4	357	1	1
	Early Summer	1267	683.5	16	72.9	79	200.8	362	1	1
	Summer	1207	666.9	16	72.7	76	125.9	344	1	1
Sault Ste. Marie	Spring	1056	720.3	16	49.7	73	125.2	331	1	1
	Early Summer	1274	844.4	16	69.5	77	140.6	345	1	1
	Summer	1262	651.9	16	52.0	72	124.5	330	1	1
Peshu Lake	Spring	893	487.7	16	69.0	79	187.9	357	1	1
	Early Summer	1274	869.4	16	87.9	78	229.2	359	1	1
	Summer	1186	693.5	16	90.2	78	142.5	345	1	1
Espanola	Spring	1016	681.7	16	62.0	75	141.7	341	1	1
	Early Summer	1274	784.9	16	65.1	78	200.6	359	1	1
	Summer	1268	838.4	16	77.9	74	151.5	338	1	1
Parry Sound	Spring	1085	697.5	16	64.8	75	151.1	343	1	1
	Early Summer	1274	730.1	16	123.1	78	231.2	365	2	2
	Summer	1274	605.9	16	93.3	78	204.6	356	1	1
Whitney	Spring	1072	701.4	16	67.8	77	208.3	355	1	1
	Early Summer	1274	782.2	16	97.1	79	174.7	359	1	1
	Summer	1273	687.6	16	59.5	77	168.0	339	1	1
Bancroft	Spring	1081	728.2	16	68.9	78	214.9	355	1	1
	Early Summer	1274	904.6	16	91.0	78	226.0	359	1	1
	Summer	1274	722.9	16	78.3	78	211.6	357	1	1

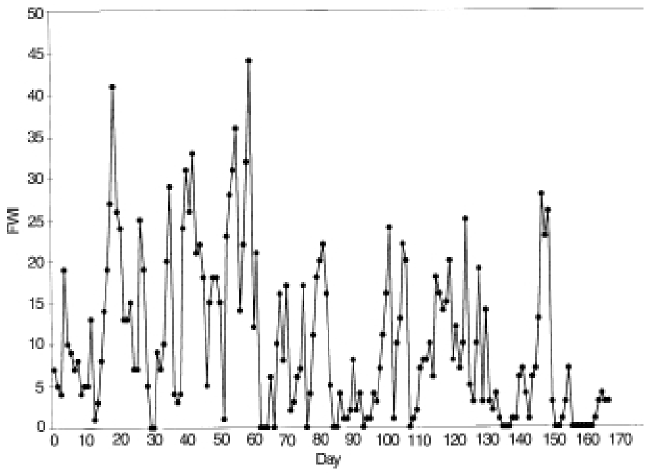

Figure 5. Daily observations of the FWI at the Kenora fire weather station during the period 13 April to 27 September 1988.

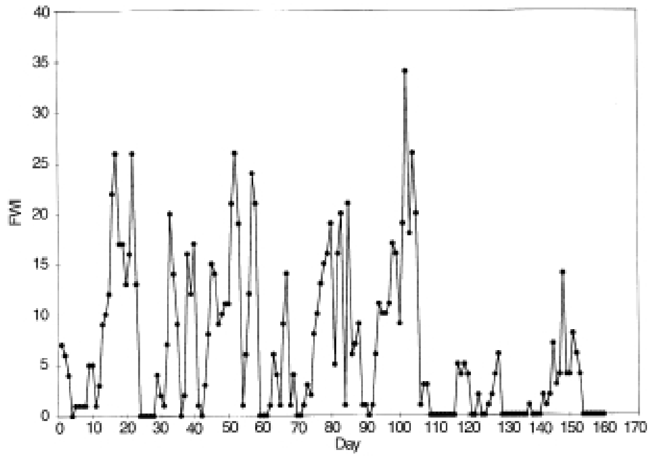

Figure 7. Daily observations of the FWI at the Shebandowan fire weather station during the period 13 April to 27 September 1988.

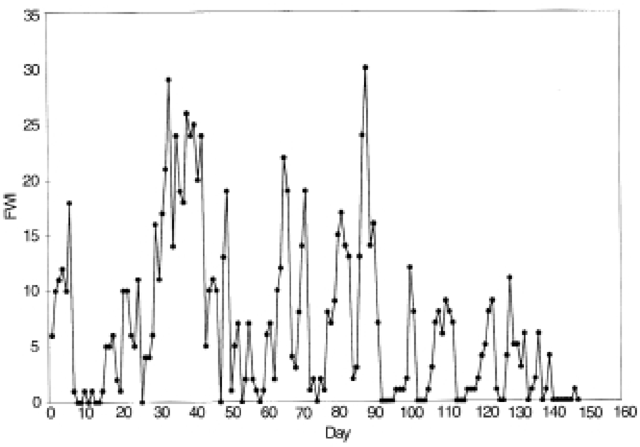

Figure 6. Daily observations of the FWI at the Kirkland Lake fire weather station during the period 13 April to 27 September 1988.

$$\hat{P}_{ijk} = n_{ijk} / \sum_{k} n_{ijk}$$

$$\hat{P}_{jk} = n_{jk} / \sum_{k} n_{jk}$$

d = number of $\hat{P}_{jk}$ values that equal zero.

If $S_{1,2}$ is greater then χ^2_α with $m(m–1)^2–d$ degrees of freedom, H_O is rejected and the third step is taken. If H_O is not rejected it is reasonable to use a Markov chain of order 1 (i.e. to assume that tomorrow's FWI class depends only on today's FWI class) and terminate the procedure.

The third step considered the hypotheses H_O and H_A

H_O : Markov chain is of order 2 or $P_{ijkl} = P_{jkl}$

H_A : Markov chain is of order 3.

The likelihood ratio test statistic $S_{2,3}$ is

$$S_{2,3} = 2 \sum_{ijkl} n_{ijkl} [\ln \hat{P}_{ijkl} – \ln \hat{P}_{jkl}]$$

and has a χ^2 distribution with $m^2(m–1)^2–d$ degrees of freedom where

n_{ijkl} = number of transitions from state i to state j to state k and then to state l;

$\hat{P}_{ijkl} = n_{ijkl} / \sum_{l} n_{ijkl}$

$\hat{P}_{jkl} = n_{jkl} / \sum_{l} n_{ijkl}$; and

d = number of $\hat{P}_{jkl}$ values that equal zero.

If $S_{2,3}$ is greater then χ^2_α with $m^2(m–1)^2–d$ degrees of freedom, H_O is rejected and it is concluded that the Markov chain is of order 3 or more. That did not happen with the fire weather data that were tested.

Results

The results of the analysis of 26 years (1963–88) of fire weather data observed at 15 fire weather stations across the province of Ontario are presented in Table 3. The first and second columns contain the names of the fire weather stations and the subseasons under investigation. The third column indicates how many 2-day transitions were included in each historical data set that was analysed. The next six columns contain the likelihood ratio test statistics and their corresponding degrees of freedom, for testing hypothesis concerning the order of the Markov chain. The null hypothesis that the Markov chain is of order $r – 1$ is rejected if the test statistic ($S_{r–1,r}$) is greater than the critical value (the χ^2

Table 4. The maximum likelihood estimates of the first order Markov chain transition probabilities for the spring, early summer and summer subseasons at Kenora.

Today's FWI Class	Tomorrow's FWI Class				
	Nil	Low	Moderate	High	Extreme
Spring					
Nil	0.595	0.123	0.153	0.101	0.028
Low	0.489	0.289	0.105	0.095	0.021
Moderate	0.135	0.286	0.385	0.175	0.020
High	0.013	0.086	0.332	0.448	0.121
Extreme	0.000	0.011	0.144	0.367	0.478
Early Summer					
Nil	0.575	0.118	0.172	0.109	0.026
Low	0.453	0.243	0.148	0.123	0.033
Moderate	0.104	0.343	0.367	0.167	0.018
High	0.015	0.066	0.318	0.505	0.096
Extreme	0.000	0.060	0.149	0.567	0.224
Summer					
Nil	0.626	0.146	0.141	0.074	0.013
Low	0.357	0.361	0.206	0.051	0.025
Moderate	0.111	0.307	0.364	0.193	0.024
High	0.004	0.072	0.378	0.454	0.092
Extreme	0.000	0.016	0.111	0.524	0.349

Table 5. The maximum likelihood estimates of the first order Markov chain transition probabilities for the spring, early summer and summer subseasons at Shebandowan.

Today's FWI Class	Tomorrow's FWI Class				
	Nil	Low	Moderate	High	Extreme
Spring					
Nil	0.726	0.222	0.047	0.004	0.000
Low	0.255	0.300	0.395	0.050	0.000
Moderate	0.214	0.085	0.487	0.210	0.004
High	0.121	0.064	0.210	0.503	0.102
Extreme	0.167	0.111	0.056	0.611	0.056
Early Summer					
Nil	0.646	0.275	0.075	0.002	0.002
Low	0.256	0.292	0.411	0.040	0.000
Moderate	0.191	0.112	0.462	0.227	0.008
High	0.138	0.115	0.264	0.391	0.092
Extreme	0.154	0.077	0.077	0.462	0.231
Summer					
Nil	0.704	0.277	0.019	0.000	0.000
Low	0.288	0.385	0.297	0.029	0.000
Moderate	0.221	0.158	0.481	0.137	0.004
High	0.147	0.073	0.257	0.477	0.046
Extreme	0.125	0.125	0.000	0.500	0.250

Table 6. The maximum likelihood estimates of the first order Markov chain transition probabilities for the spring, early summer and summer subseasons at Kirkland Lake

Today's FWI Class	Tomorrow's FWI Class				
	Nil	Low	Moderate	High	Extreme
Spring					
Nil	0.658	0.305	0.032	0.003	0.003
Low	0.200	0.414	0.340	0.039	0.007
Moderate	0.214	0.162	0.428	0.183	0.013
High	0.135	0.095	0.159	0.492	0.119
Extreme	0.088	0.147	0.029	0.324	0.412
Early Summer					
Nil	0.594	0.350	0.048	0.008	0.000
Low	0.214	0.423	0.314	0.046	0.003
Moderate	0.146	0.152	0.464	0.226	0.012
High	0.141	0.131	0.180	0.510	0.039
Extreme	0.095	0.095	0.238	0.333	0.238
Summer					
Nil	0.705	0.274	0.021	0.000	0.000
Low	0.261	0.442	0.272	0.025	0.000
Moderate	0.192	0.184	0.500	0.117	0.008
High	0.150	0.100	0.275	0.438	0.038
Extreme	0.111	0.000	0.222	0.333	0.333

value for $\alpha = 0.05$ or 0.01 and the corresponding degrees of freedom). The last two columns show the Markov chain rank that it is reasonable to use for each weather station and subseason given the statistical test results, for the $\alpha = 0.05$ and $\alpha = 0.01$ levels of significance. The statistical test results presented in Table 3 can be summarized as follows.

1. Spring: The $\alpha = 0.01$ test results indicate that it is reasonable to use a Markov chain of order 1 at all 15 weather stations. The $\alpha = 0.05$ test results suggest a Markov chain of order 1 for 14 of the 15 stations, and one of order 2 for Red Lake.
2. Early Summer: The $\alpha = 0.01$ test results indicate it is reasonable to use a Markov chain of order 1 at 14 of the 15 weather stations, and one of order 2 for Parry Sound. The $\alpha = 0.05$ test results also suggest a Markov chain of order 1 for 14 of the 15 stations and one of order 2 for Parry Sound.
3. Summer: The $\alpha = 0.01$ test results indicate it is reasonable to use a Markov chain of order 1 at 13 of the 15 weather stations, and Markov chains of order 2 for Chapleau and Kenora. The $\alpha = 0.05$ test results suggest a Markov chain of order 1 for 12 of the 15 stations, and Markov chains of order 2 for Chapleau, Geraldton, and Kenora.

Note however, that the likelihood ratio test is an asymptotic test and the sample size is very low given the number of parameters to be estimated for a Markov chain of order 3.

It is difficult to visualize the Markovian properties of the FWI by studying a time series graph of 26 years of data with 153 observations each year. We therefore plotted the raw FWI observed at Kenora, Shebandowan, and Kirkland Lake for a single year. We arbitrarily chose 1988, the last year included in the sample. The results are presented in Figures 5, 6, and 7. The corresponding maximum likelihood estimates of the first order Markov chains for the spring, early summer and summer subseasons are presented in Tables 4, 5, and 6.

Discussion

A Markov chain of order 1 and 5 FWI classes leads to a Markov transition matrix with 25 elements to be estimated. The number of elements to be estimated coupled with the statistical test results and the mathematical structure of the CFFWIS suggest that it is reasonable to limit modelling of the day to day changes in the FWI class in Ontario to Markov chains of order 1. That is consistent with the results presented by Greulich (1976) who opted to use a first order Markov chain to model day to day changes in a brush burning index despite the fact that his likelihood ratio test results suggested a second order model might have been more appropriate for one of two fire weather stations located in the California Division of Forestry's District 1.

There are several issues that should be addressed in future studies. First, since fire management system models are by no means limited to the use of the FWI, the applicability of Markov models for the other CFFWIS indices and even basic fire weather observations such as precipitation should also be investigated. Second, the question of time homogeneity should be more carefully investigated with a view to developing simple procedures to cope with non-homogeneous transition probabilities, particularly during the summer subseason.

The Fire Weather Index itself is a continuous variable. Fujioka and Tsou (1985) reported preliminary results concerning their application of time series techniques to day to day changes in a fire danger rating index based on more than 3000 weather observations recorded at four weather stations in California. Their preliminary results suggested a second order autoregressive process model might be appropriate. Further effort should be devoted to investigations of the extent to which time series analysis methods (see for example Box and Jenkins 1976 and Chatfield 1989) can be used to model day to day variations in fire weather and comparing such approaches with the use of Markov chains. Time series analysis might also provide additional insight that could be used to help decide how to partition the fire season into subseasons and the order of the Markov chain within each subseason.

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