

Topological Data Analysis

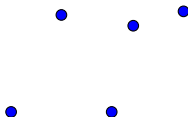
Rips, Čech and Morse filtrations

Motiejus Valiunas

IM UW, Summer Semester 2022/23

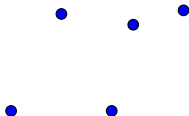
9 March 2023

Point cloud:

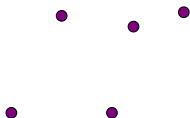


Rips and Čech filtrations

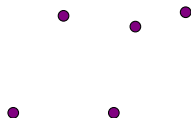
$$i = 0$$



Čech complex K_i

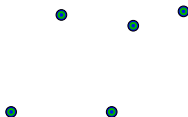


Rips complex K_i

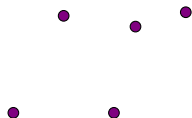


Rips and Čech filtrations

$$i = 1$$

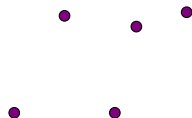


Čech complex K_i



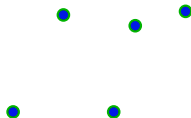
$i \in$

Rips complex K_i

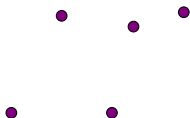


Rips and Čech filtrations

$$i = 2$$

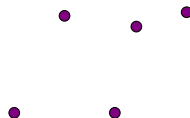


Čech complex K_i



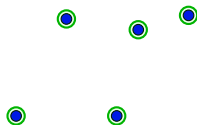
$i \in \mathcal{E}$

Rips complex K_i

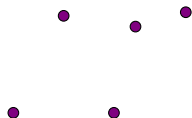


Rips and Čech filtrations

$$i = 3$$

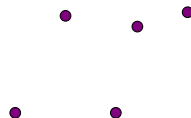


Čech complex K_i



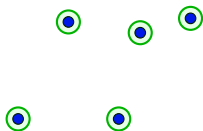
$\leftrightarrow$
 $i\mathcal{E}$

Rips complex K_i

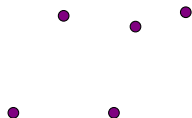


Rips and Čech filtrations

$$i = 4$$

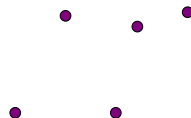


Čech complex K_i



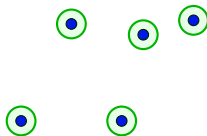
$\leftrightarrow$
 $i\mathcal{E}$

Rips complex K_i

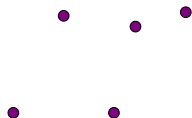


Rips and Čech filtrations

$$i = 5$$

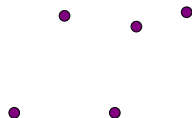


Čech complex K_i



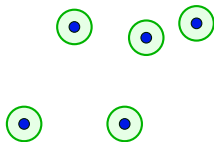
$\longleftrightarrow$
 $i\epsilon$

Rips complex K_i

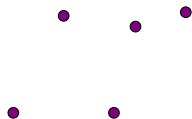


Rips and Čech filtrations

$$i = 6$$

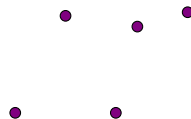


Čech complex K_i



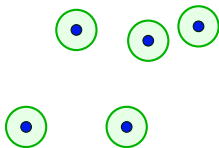
$\longleftrightarrow$
 $i\varepsilon$

Rips complex K_i

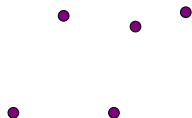


Rips and Čech filtrations

$$i = 7$$

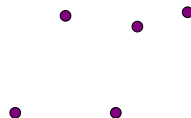


Čech complex K_i



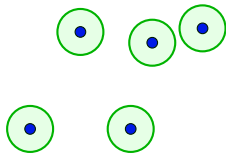
$\longleftrightarrow$
 $i\varepsilon$

Rips complex K_i

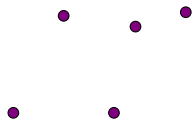


Rips and Čech filtrations

$$i = 8$$

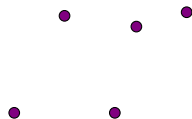


Čech complex K_i



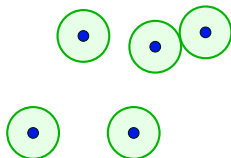
$\longleftrightarrow$
 $i\epsilon$

Rips complex K_i

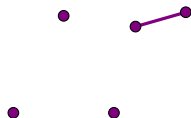


Rips and Čech filtrations

$$i = 9$$



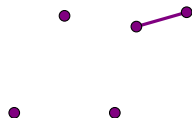
Čech complex K_i



$$\longleftrightarrow$$

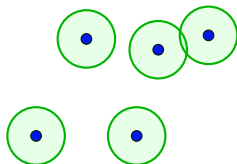
$$i\varepsilon$$

Rips complex K_i

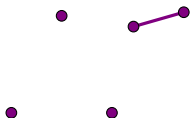


Rips and Čech filtrations

$$i = 10$$



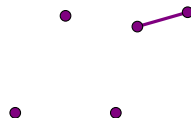
Čech complex K_i



$$\longleftrightarrow$$

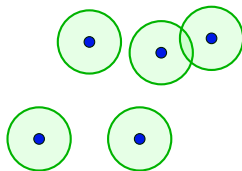
$$i\varepsilon$$

Rips complex K_i

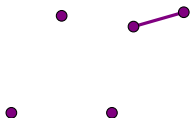


Rips and Čech filtrations

$$i = 11$$



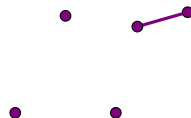
Čech complex K_i



$$\longleftrightarrow$$

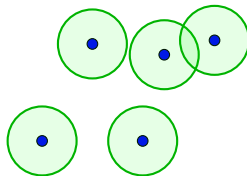
$i\varepsilon$

Rips complex K_i

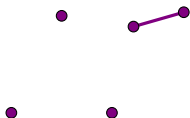


Rips and Čech filtrations

$$i = 12$$

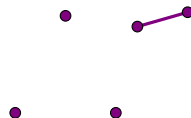


Čech complex K_i



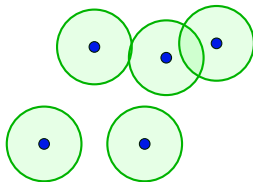
$\longleftrightarrow$
 $i\epsilon$

Rips complex K_i

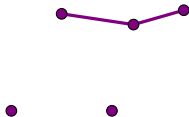


Rips and Čech filtrations

$$i = 13$$



Čech complex K_i



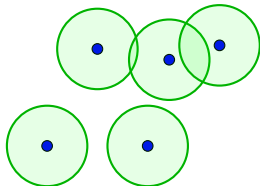
$\longleftrightarrow$
 $i\epsilon$

Rips complex K_i

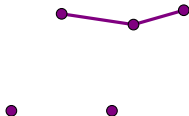


Rips and Čech filtrations

$$i = 14$$



Čech complex K_i

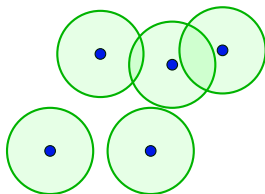


Rips complex K_i

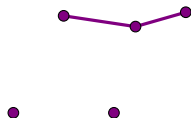


Rips and Čech filtrations

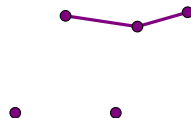
$$i = 15$$



Čech complex K_i

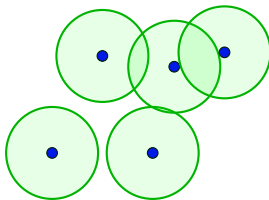


Rips complex K_i

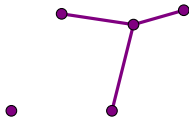


Rips and Čech filtrations

$$i = 16$$

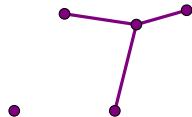


Čech complex K_i



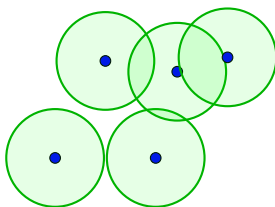
$$\longleftrightarrow i\epsilon$$

Rips complex K_i

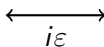
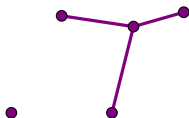


Rips and Čech filtrations

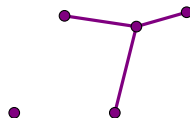
$$i = 17$$



Čech complex K_i

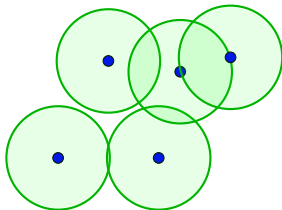


Rips complex K_i

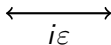
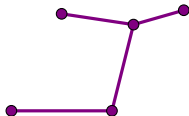


Rips and Čech filtrations

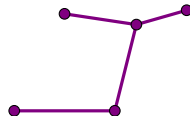
$$i = 18$$



Čech complex K_i

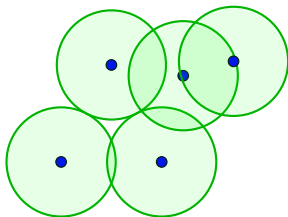


Rips complex K_i

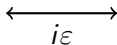
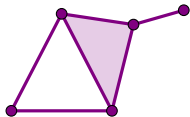


Rips and Čech filtrations

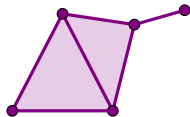
$$i = 19$$



Čech complex K_i

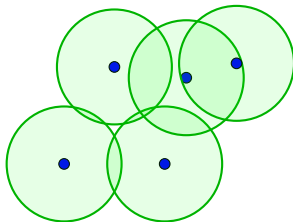


Rips complex K_i

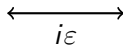
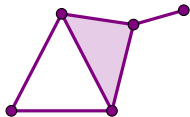


Rips and Čech filtrations

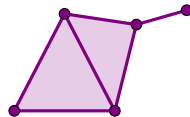
$$i = 20$$



Čech complex K_i

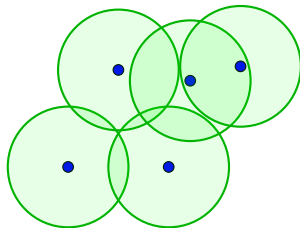


Rips complex K_i

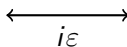
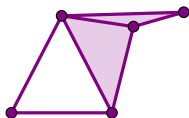


Rips and Čech filtrations

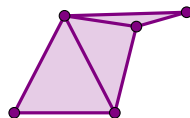
$$i = 21$$



Čech complex K_i

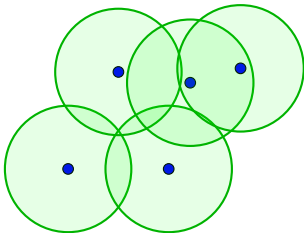


Rips complex K_i

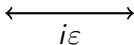
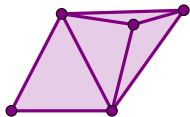


Rips and Čech filtrations

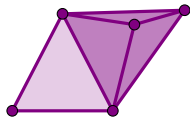
$$i = 22$$



Čech complex K_i

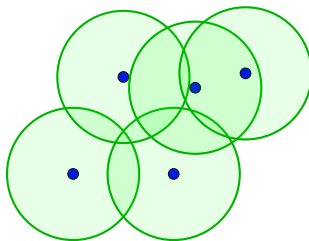


Rips complex K_i

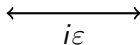
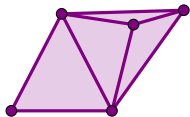


Rips and Čech filtrations

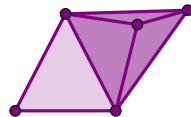
$$i = 23$$



Čech complex K_i

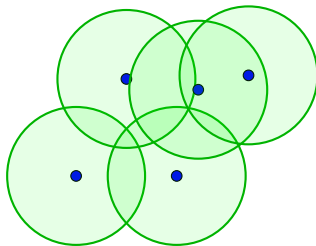


Rips complex K_i

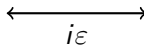
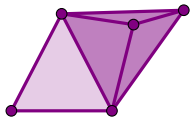


Rips and Čech filtrations

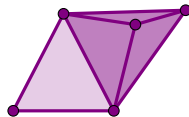
$$i = 24$$



Čech complex K_i

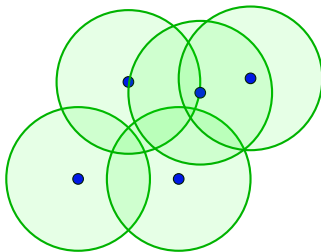


Rips complex K_i

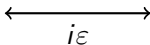
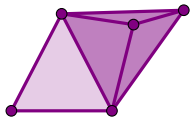


Rips and Čech filtrations

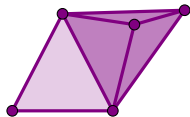
$$i = 25$$



Čech complex K_i

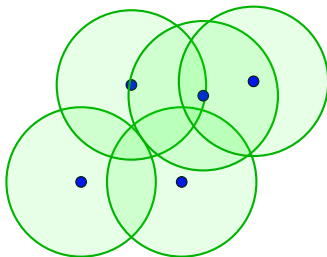


Rips complex K_i

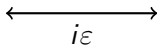
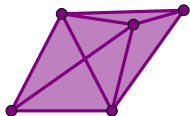


Rips and Čech filtrations

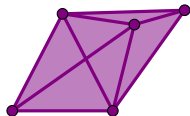
$$i = 26$$



Čech complex K_i

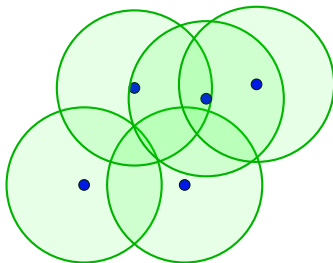


Rips complex K_i

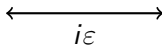
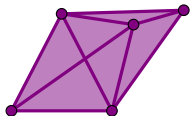


Rips and Čech filtrations

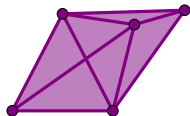
$$i = 27$$



Čech complex K_i

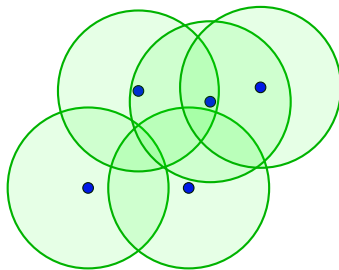


Rips complex K_i

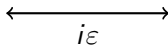
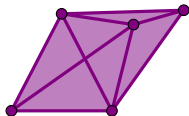


Rips and Čech filtrations

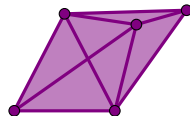
$$i = 28$$



Čech complex K_i

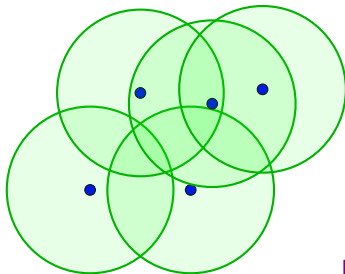


Rips complex K_i

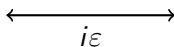
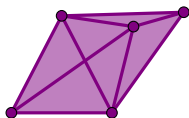


Rips and Čech filtrations

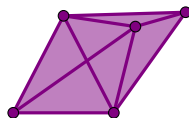
$$i = 29$$



Čech complex K_i

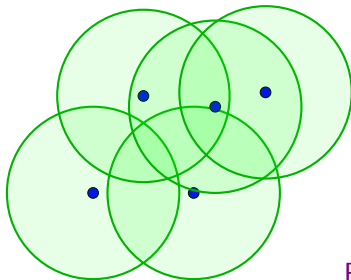


Rips complex K_i

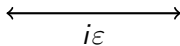
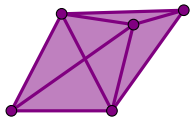


Rips and Čech filtrations

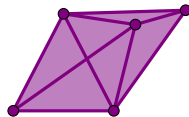
$$i = 30$$



Čech complex K_i

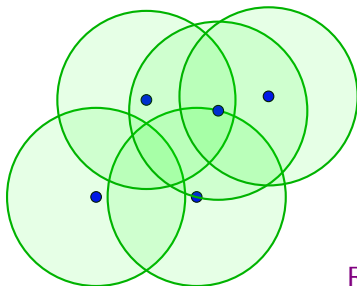


Rips complex K_i

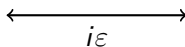
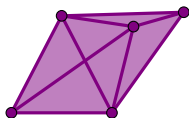


Rips and Čech filtrations

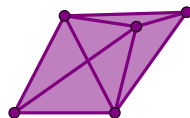
$$i = 31$$



Čech complex K_i

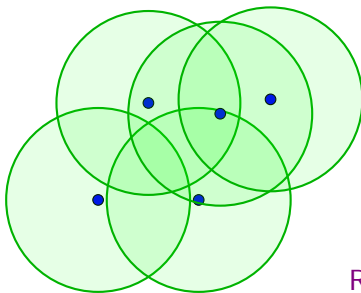


Rips complex K_i

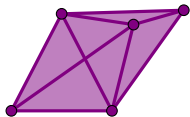


Rips and Čech filtrations

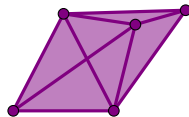
$$i = 32$$



Čech complex K_i

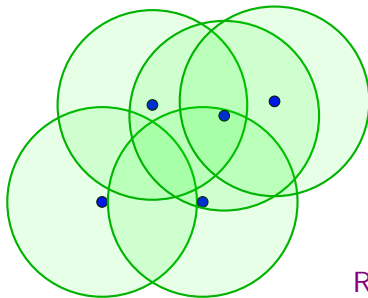


Rips complex K_i

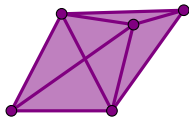


Rips and Čech filtrations

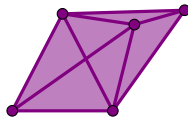
$$i = 33$$



Čech complex K_i

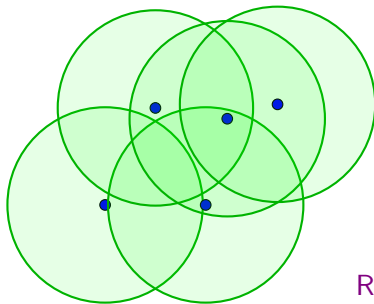


Rips complex K_i

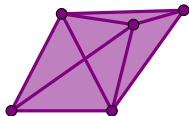


Rips and Čech filtrations

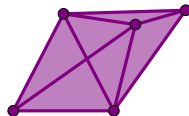
$$i = 34$$



Čech complex K_i

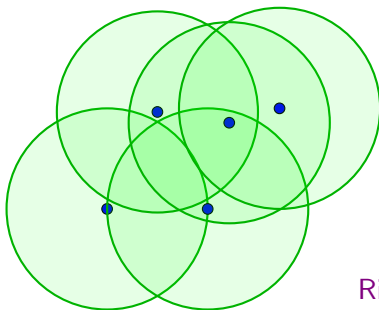


Rips complex K_i

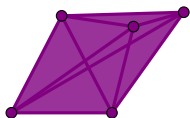


Rips and Čech filtrations

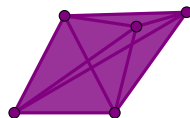
$$i = 35$$



Čech complex K_i

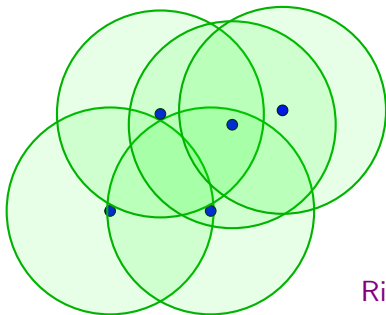


Rips complex K_i

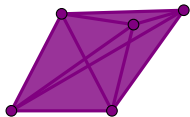


Rips and Čech filtrations

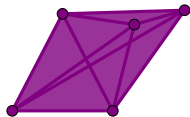
$$i = 36$$



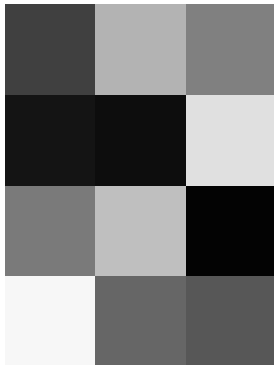
Čech complex K_i



Rips complex K_i

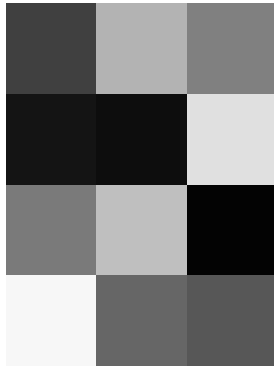


Picture:



Morse filtration

$$i = 0$$

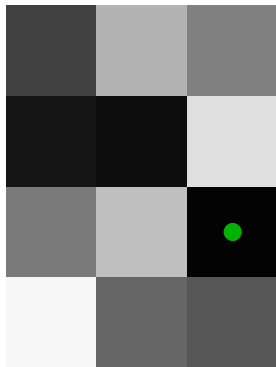


Morse complex K_i

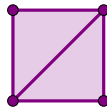


Morse filtration

$$i = 1$$



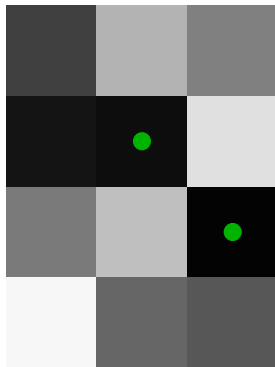
Morse complex K_i



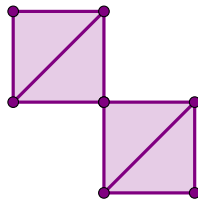
↑
 $i\epsilon$

Morse filtration

$$i = 2$$

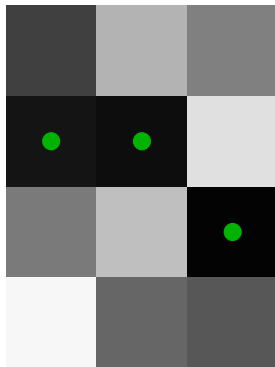
 $i\epsilon$

Morse complex K_i



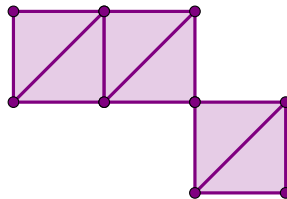
Morse filtration

$$i = 3$$



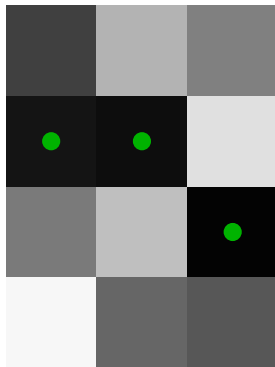
$i\epsilon$

Morse complex K_i

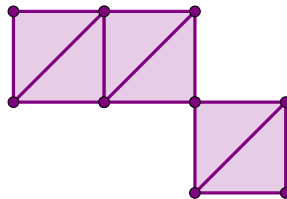


Morse filtration

$$i = 4$$

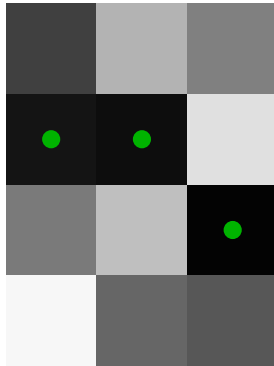
 $i\epsilon$

Morse complex K_i



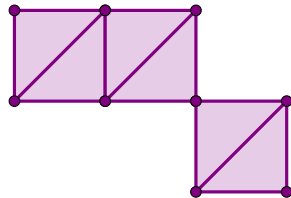
Morse filtration

$$i = 5$$



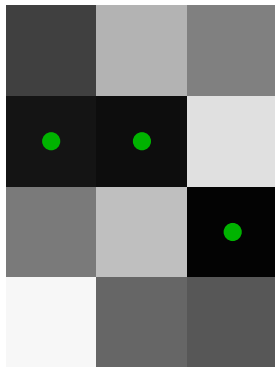
$\uparrow$
 $i\epsilon$

Morse complex K_i



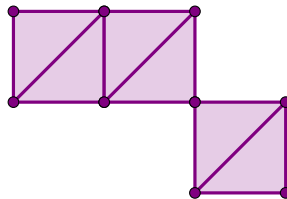
Morse filtration

$$i = 6$$



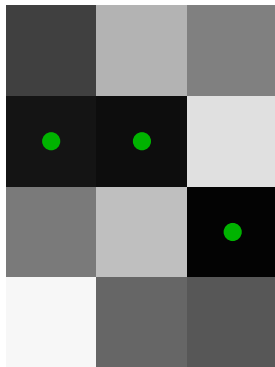
↑
 $i\epsilon$

Morse complex K_i



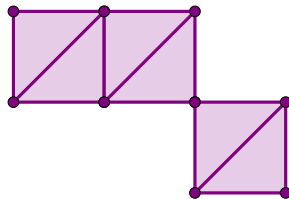
Morse filtration

$$i = 7$$



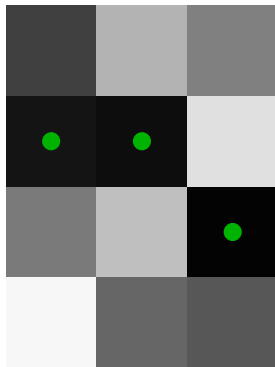
$\uparrow$
 $i\epsilon$

Morse complex K_i



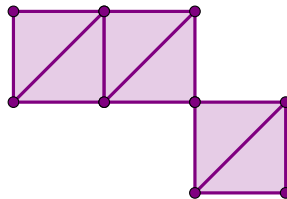
Morse filtration

$$i = 8$$



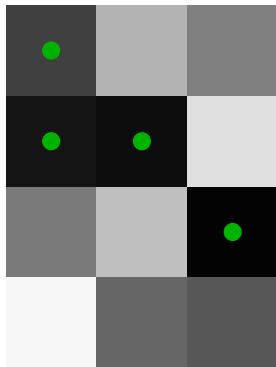
↑
 $i\epsilon$

Morse complex K_i

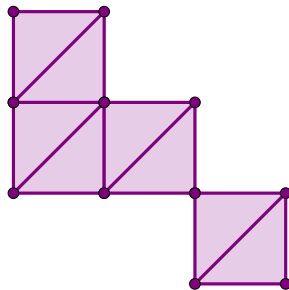


Morse filtration

$$i = 9$$

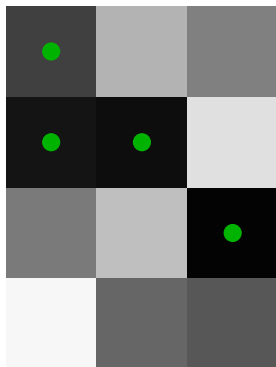
 $i\epsilon$

Morse complex K_i

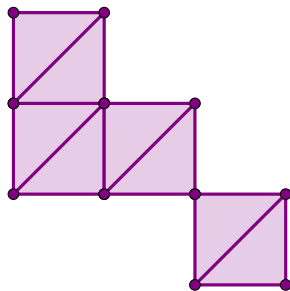


Morse filtration

$$i = 10$$

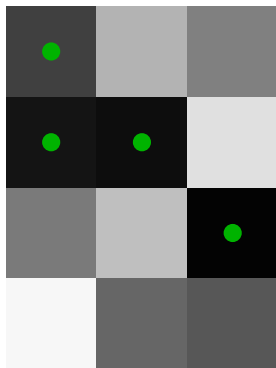
 $i\epsilon$

Morse complex K_i



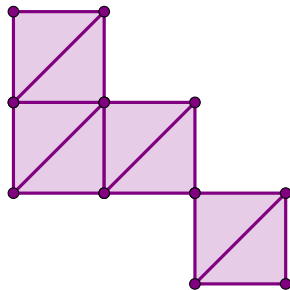
Morse filtration

$$i = 11$$



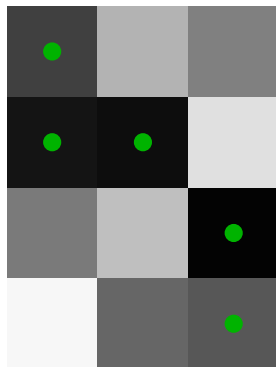
↑
 $i\epsilon$

Morse complex K_i



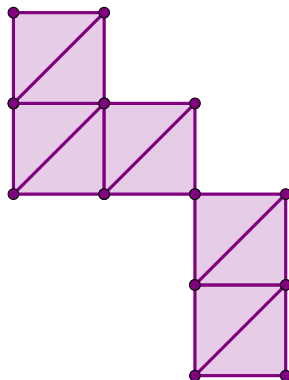
Morse filtration

$$i = 12$$



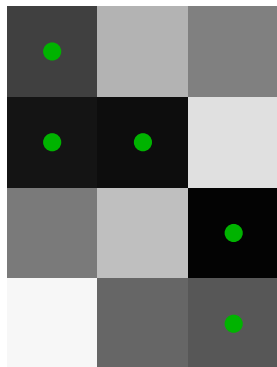
$\uparrow$
 $i\epsilon$

Morse complex K_i



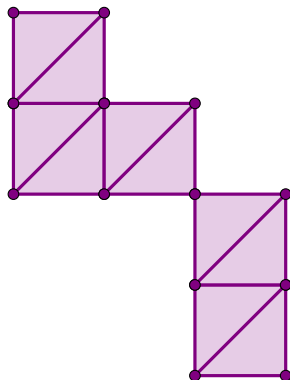
Morse filtration

$$i = 13$$



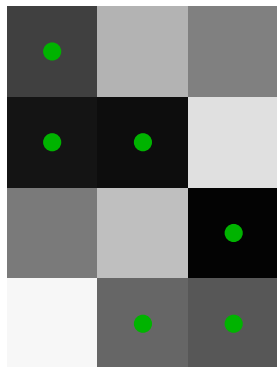
$\uparrow$
 $i\epsilon$

Morse complex K_i



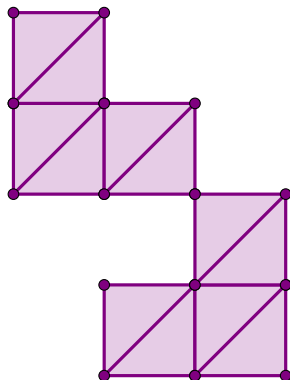
Morse filtration

$$i = 14$$



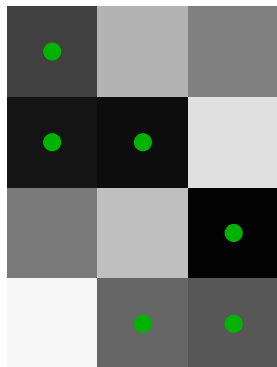
$\uparrow$
 $i\epsilon$

Morse complex K_i



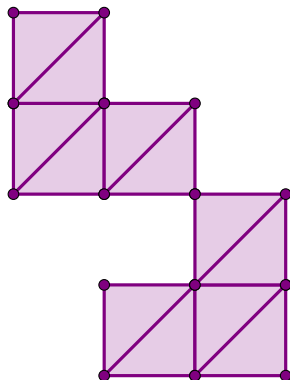
Morse filtration

$$i = 15$$



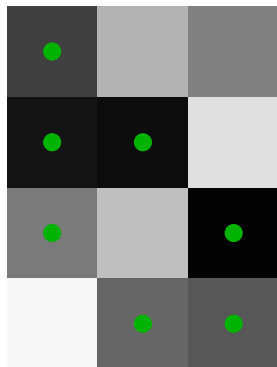
$\uparrow$
 $i\epsilon$

Morse complex K_i



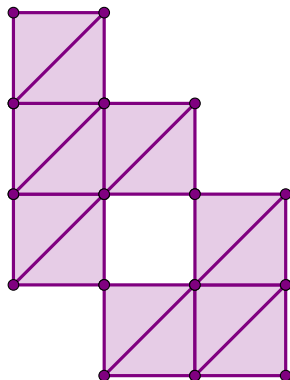
Morse filtration

$$i = 16$$



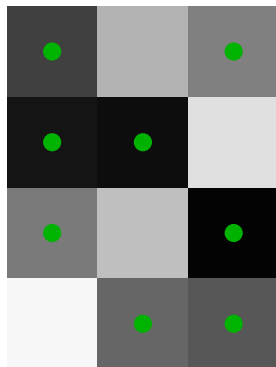
$\uparrow$
 $i\epsilon$

Morse complex K_i



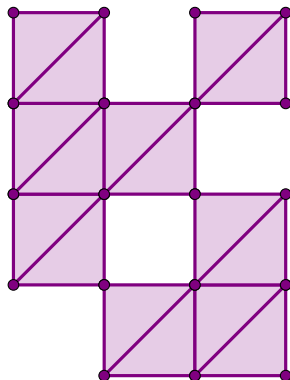
Morse filtration

$i = 17$



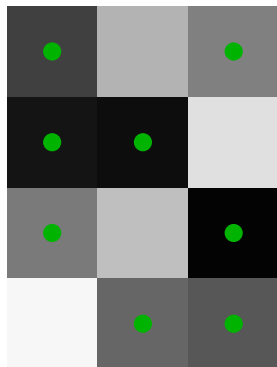
$\uparrow$
 $i\epsilon$

Morse complex K_i



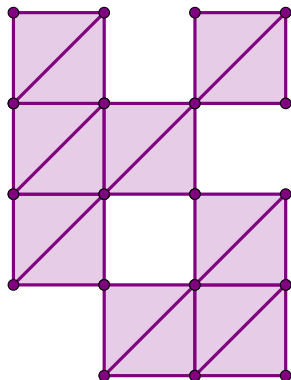
Morse filtration

$$i = 18$$



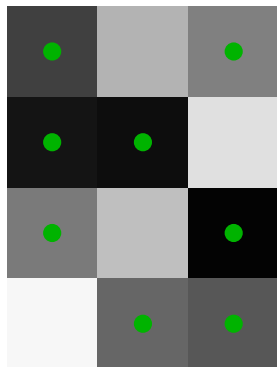
$\uparrow$
 $i\epsilon$

Morse complex K_i



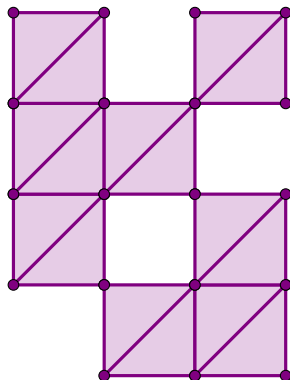
Morse filtration

$$i = 19$$



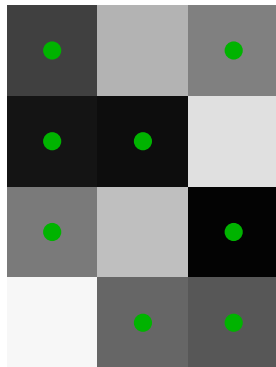
$\uparrow$
 $i\epsilon$

Morse complex K_i



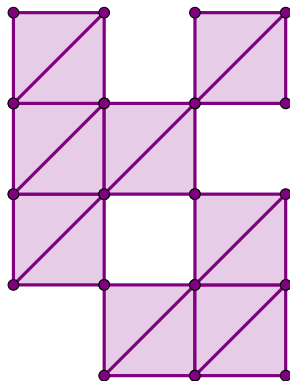
Morse filtration

$$i = 20$$



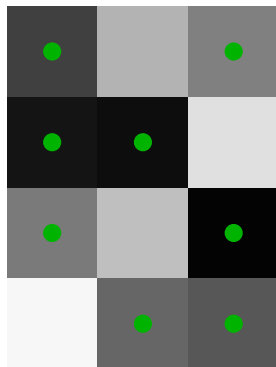
$\uparrow$
 $i\epsilon$

Morse complex K_i



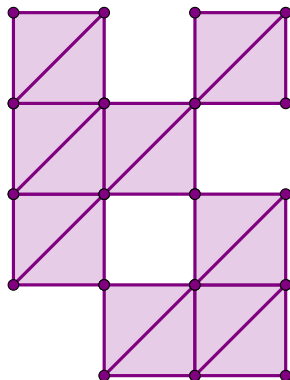
Morse filtration

$i = 21$



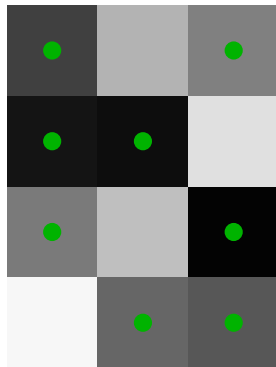
$\uparrow$
 $i\epsilon$

Morse complex K_i



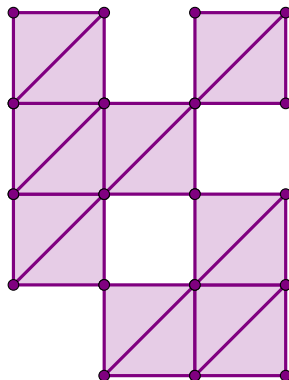
Morse filtration

$$i = 22$$



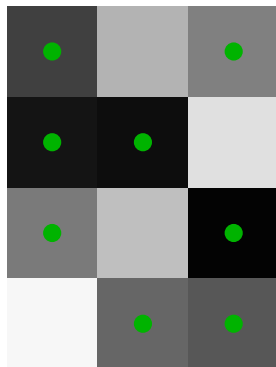
$\uparrow$
 $i\epsilon$

Morse complex K_i



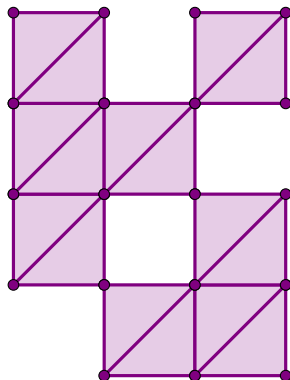
Morse filtration

$$i = 23$$



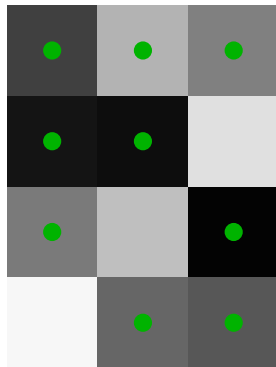
$\uparrow$
 $i\epsilon$

Morse complex K_i



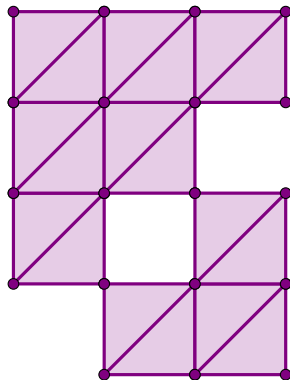
Morse filtration

$$i = 24$$



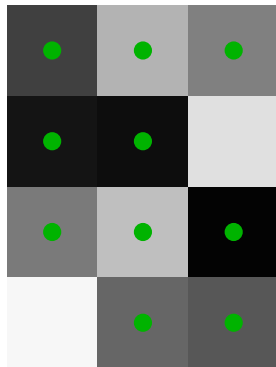
$\uparrow$
 $i\epsilon$

Morse complex K_i



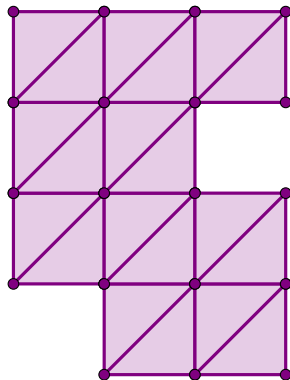
Morse filtration

$$i = 25$$



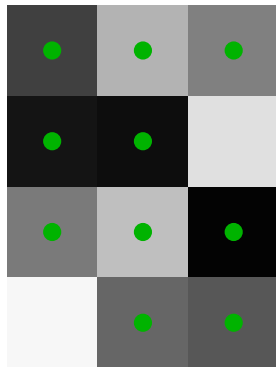
$\uparrow$
 $i\epsilon$

Morse complex K_i



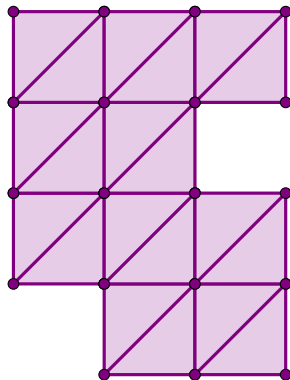
Morse filtration

$$i = 26$$



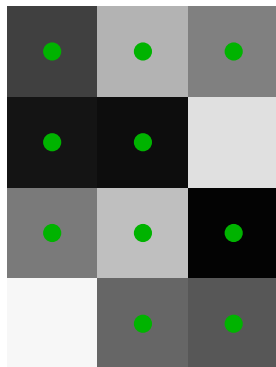
$i\epsilon$

Morse complex K_i



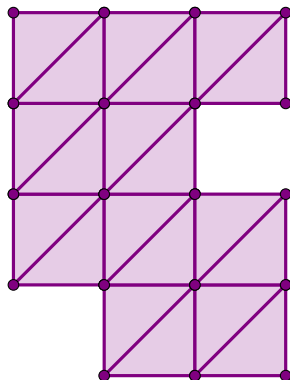
Morse filtration

$i = 27$



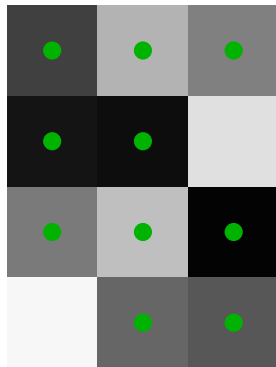
$\uparrow$
 $i\epsilon$

Morse complex K_i



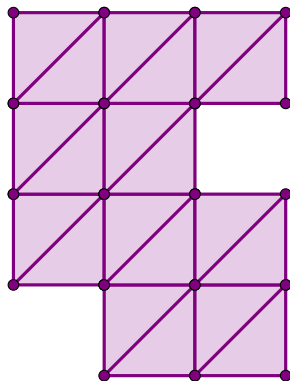
Morse filtration

$$i = 28$$



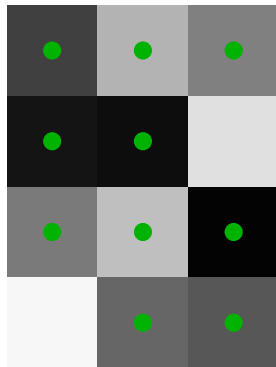
$\uparrow$
 $i\epsilon$

Morse complex K_i



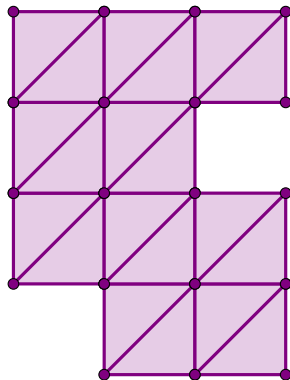
Morse filtration

$$i = 29$$



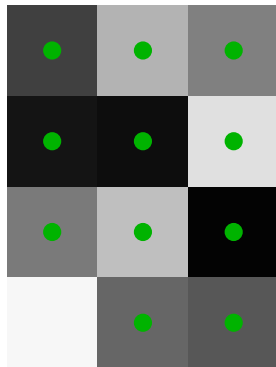
$\uparrow$
 $i\epsilon$

Morse complex K_i

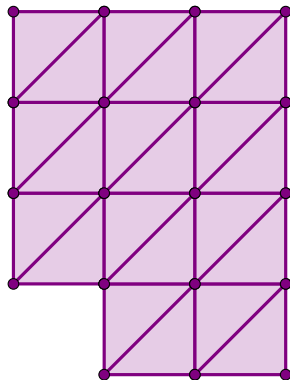


Morse filtration

$$i = 30$$

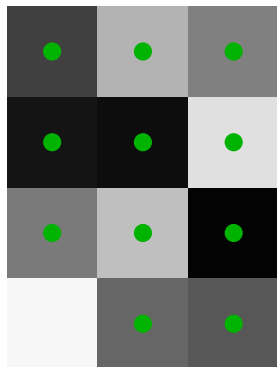


Morse complex K_i

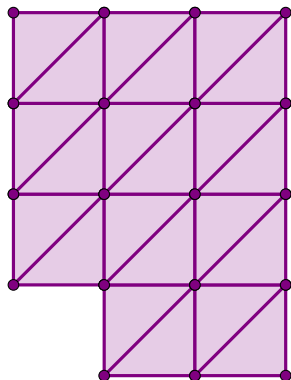


Morse filtration

$$i = 31$$

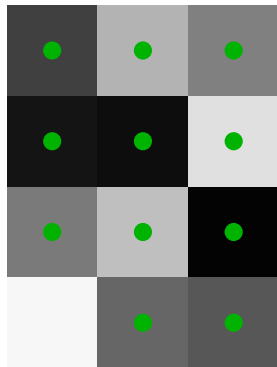


Morse complex K_i

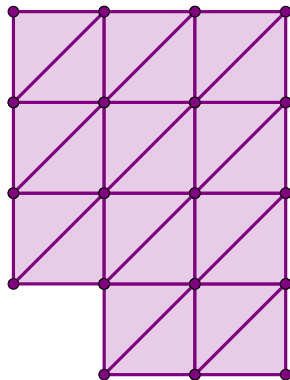


Morse filtration

$$i = 32$$

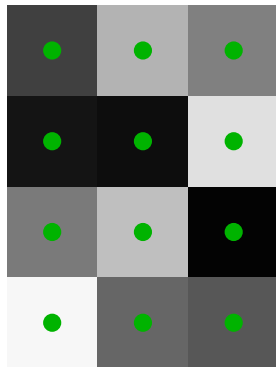


Morse complex K_i



Morse filtration

$$i = 33$$



Morse complex K_i

