Egzam A2/Rzeszotnik/04 luty 2019/first part - 80 minutes

Name:

1. Calculate the integral

$$\int \ln(\sqrt{x}+1)\,dx$$

Hint: Use a substitution $\sqrt{x}=t$, and after that use integration by parts.

2. Calculate the limit

$$\lim_{n\to\infty}\left(\frac{1}{n^2+1}+\frac{2}{n^2+4}+\frac{3}{n^2+9}+\cdots+\frac{n}{n^2+n^2}\right)$$

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3. Calculate the volume of a solid obtained by the revolution about the OX axis of the region bounded by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \qquad \text{where } a, b > 0.$$

Remainder: Such a volume is given by $V = \pi \int_c^d f^2(x)dx$.

4. Find all the values of the parameter $p \in \mathbb{R}$ such that the series given below is convergent

$$\sum_{n=1}^{\infty} \left(\int_0^{\frac{1}{n}} x^p dx \right)$$