

Lista 4-1. Ciągi i granice.

W poniższych zadaniach należy obliczyć granice podanych ciągów, lub stwierdzić, że ta granica nie istnieje.

61. $2^n - \frac{1}{n}, \quad \frac{2}{\log_2(n+1)}, \quad n \cdot (-1)^n.$

62. $\frac{5n^3 + n^2 - 6}{3n^4 + 7}, \quad \frac{5n^4 + n^2 - 6}{3n^4 + 7}, \quad \frac{5n^5 + n^2 - 6}{3n^4 + 7}.$

63. $\frac{1+2+3+\dots+n}{n^2}, \quad \frac{1+2+4+\dots+2^n}{1+3+9+\dots+3^n}, \quad \frac{1-2+3-4+5-6+\dots+(2n-1)-2n}{\sqrt{n^2+2}}.$

64. $\frac{n}{1+\sqrt{n}}, \quad \frac{\sqrt[3]{n^2+n}}{n+2}, \quad \frac{(\sqrt{n+1}+\sqrt{n})^7}{n^3(1+7\sqrt{n+2})}.$

65. $\sqrt[n]{n}, \quad \sqrt[n]{n^2}, \quad \sqrt[n]{2n^3+17n+1}.$

66. $\sqrt{n+3}-\sqrt{n}, \quad \sqrt{n^2+3n}-n, \quad n(\sqrt{n^2+7}-n).$

67. $\frac{(-1)^n}{n}, \quad \frac{1}{(2+(-1)^n)^n}, \quad a_n = \begin{cases} (-1)^n \cdot n! & \text{dla } n \leq 100 \\ \frac{2^n}{2^n+n} & \text{dla } n > 100 \end{cases}.$

68. $\frac{n^2+1}{n^3+1} + \frac{n^2+2}{n^3+2} + \frac{n^2+3}{n^3+3} + \dots + \frac{n^2+n}{n^3+n}, \quad \frac{1}{n^2} + \frac{1}{n^2+1} + \frac{1}{n^2+2} + \dots + \frac{1}{(n+1)^2}.$

69. $\frac{n^7}{7^n}, \quad \frac{10^n}{n!}, \quad \frac{n!}{n^{22}}.$

70. $\frac{\sqrt{3^n+2^n}}{\sqrt{3^n+1}}, \quad \frac{\sqrt{3^n+1}}{\sqrt{2^n+n^2}}, \quad \frac{\sqrt{2^n+2}}{\sqrt[3]{3^n+3}}.$