Random Integral Representations: THE CONJECTURE

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In my article in THE ANNALS OF PROBABILITY, vol. **13**, No 2, 1985 on page 607 I had stated the following conjecture:

Each class of limit laws, derived from sequences of independent random variables, is the image of some subset of ID (infinite divisible) by some mapping defined as random integral.

[It was repeated in PROBABILITY THEORY AND RELATED FIELDS, vol. **78**, 1988, page 474]

Added on September 18, 2004.

Although I have no formal proof of that general claim such representations were found for many specific classes of laws, Furthermore, twenty years later I may "specify" that the representations in question might be of the form

$$\mathcal{L}\big(\int_A h(t)dY(r(t))\big) \tag{1}$$

where A is a subset of the real (positive) line, $Y(\cdot)$ is a Lévy process (possibly with some moments restrictions) and h, r are some deterministic functions and the later one as an "inner time change" must be monotonic. ($\mathcal{L}(\cdot)$ denotes the probability distributions of a random variable (here it is given as random integral).)

Added on June 25, 2011.

A review, until 2010, of the research where random integral representations (1) had appeared, is presented in the invited talk at the Vilnius Conference 2010; cf. *Lithuanian Mathematical Journal*, vol. **51**, no 3, 2011, pp. 692-698. (arXiv.1009.2418 [math PR])

Added on May 15, 2016.

In recent papers with Richard C. Bradley (for example cf. *Journal of Theoretical Probability*, **29**, March 2016, pp. 292-306), we proved representations like the above (1) for limit laws of *strongly mixing sequences* but not necessarily stochastically independent random variables. (arXiv.1403.1441 [Math. PR]). This suggests that the Conjecture may be true in more general case.