JUREK COORDINATES

This mathematical term appeared in some papers and recently in the mongraph *STOCHASTIC MODELS FOR FRACTIONAL CALCULUS* by Mark M. Meerschaert and Alla Sikorskii, de Gruyter Studies in Mathematics 43, 2012, page 185.

There, it is a particular case of s o called *polar coordinates* in Banach spaces; cf.[1]. Namely,

PROPOSITION 1. *For an one-parameter multipicative group* $(T_t, t > 0)$ *, of bounded linear operators (on a Banach space X), satisfying the condition* $\lim_{t\to 0} T_t x = 0, x \in X$*, there exists an equivalent norm* $|||.|||$ *and a Borel set* $S \subset X \setminus \{0\}$ *such that*

(i) *the mapping* $(0,\infty) \times S \ni (t,s) \to T_t s \in X \setminus \{0\}$
is one-to-one, onto, continuous and its inverse is Borel measurable;
(ii) *mappings* $t \to |||T_t x|||$ *are strictly increasing for* $x \in X \setminus \{0\}$.

In a case of uniformly continuous one-parmetr groups one has more:

PROPOSITION 2. *Let* $T_t = t^B, t > 0$ *(B is a bounded linear operator) be such that* $\lim_{t\to 0} t^B = 0$*. Then* $||x||_B := \int_0^1 ||t^B x|| t^{-1} dt$ *is an equivalent norm and as the set S (in Proposition 1) one takes* $S_B := \{x : ||x||_B = 1\}$.

Those polar coordinates are usueful when one has to solve measure inequalites such as $M(.) \geq (T_t M)(.)$ or measure equations as $t\,M(.) = (t^B M)(.)$. The solutions to the last equation has to have form $M(dx) = r^{-2}\,dr\,m(ds)$ when $x = r^B\,s$; cf. [4], p. 196. Also cf. [1], [2] and [3].

[1] Z. J. Jurek (**1984**), On polar coordinates in Banach spaces. *Bull. Pol. Acad.: Math.* vol. 32, pp. 61-66. (Note the open problems in Remarks.)

[2] Z. J. Jurek (**1984**), How to solve the inequality: $U_t m < m$ for every $0 < t < 1$? *Prob. Math. Stat.* vol. 4 , pp. 171-183.

[3] Z. J. Jurek (**1983**) Limit distributions and one-parameter groups of linear operators on Banach spaces. *J. Multivar. Anal.* vol. 13, pp. 578-604.

[4] Z. J. Jurek (**1993**), *Opertor-limit distributions in probability theory,* J. Wiley and Sons (1993); (co-author J. D. Mason).