JUREK COORDINATES

This mathematical term appeared in some papers and recently in the monograph *STOCHASTIC MODELS FOR FRACTIONAL CALCULUS* by Mark M. Meerschaert and Alla Sikorskii, de Gruyter Studies in Mathematics 43, 2012, page 185.

There, it is a particular case of so called polar coordinates in Banach spaces; cf.[1]. Namely,

**PROPOSITION 1.** For an one-parameter multiplicative group \((T_t, t > 0)\), of bounded linear operators ( on a Banach space \(X\)), satisfying the condition \(\lim_{t \to 0} T_t x = 0, x \in X\), there exists an equivalent norm \(||.||\) and a Borel set \(S \subset X \setminus \{0\}\) such that

(i) the mapping \((0, \infty) \times S \ni (t, s) \to T_t s \in X \setminus \{0\}\) is one-to-one, onto, continuous and its inverse is Borel measurable;

(ii) mappings \(t \to |||T_t x|||\) are strictly increasing for \(x \in X \setminus \{0\}\).

In a case of uniformly continuous one-parameter groups one has more:

**PROPOSITION 2.** Let \(T_t = t^B, t > 0\) (\(B\) is a bounded linear operator) be such that \(\lim_{t \to 0} t^B = 0\). Then \(||x||_B := \int_0^1 ||t^B x|| t^{-1} dt\) is an equivalent norm and as the set \(S\) (in Proposition 1) one takes \(S_B := \{x : ||x||_B = 1\}\).

Those polar coordinates are useful when one has to solve measure inequalities such as \(M(.) \geq (T_t M)(.)\) or measure equations as \(t M(.) = (t^B M)(.)\). The solutions to the last equation have to have form \(M(dx) = r^{-2} dr m(ds)\) when \(x = r^B s\); cf. [4], p. 196. Also cf. [1], [2] and [3].


