PROBABILITY
AND
MATHEMATICAL STATISTICS
Vol. 33, Fasc. 1 (2013), pp. 79–92

# VARIANCE REDUCTION BY SMOOTHING REVISITED

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*Abstract:* Smoothing is a common method used in nonparametric statistics and on many occasions it has been noted that it may result in an asymptotic variance reduction or increase of efficiency. Another well-known effect associated with smoothing is that it introduces a small bias. In the first part of the paper we show that if the influence function of a Hadamard-differentiable statistical functional or its derivative have jumps, then functionals of a kernel-smoothed cumulative distribution function may have lower asymptotic variance than the variance of the original functional. This extends and unifies previous results and shows detailed conditions under which the asymptotic variance reduction by smoothing can be achieved. The smoothing however introduces a small bias of order $O(h^2)$, where $h$ is a smoothing parameter. In the second part of the paper we discuss the optimal balance between the bias and variance reduction.

**2000 AMS Mathematics Subject Classification:** Primary: 62F12; Secondary: 62E20, 62G20, 62G35.

**Keywords and phrases:** Smoothing; influence curve; variance reduction; robustness; statistical functionals.