Preservation of stochastic orders under transformations defined in reliability theory

Abstract

The aim of PhD thesis was to make a systematized review of stochastic orders' preservation rules and to formulate new theorems. There was seven stochastic orders chosen as the basis: the usual, the hazard rate, the reversed hazard rate, the likelihood ratio, the dispersive, the convex transform and the star orders. Since the generalized total time on test transform (GTTT) and the GTTT transform order play an important role in the thesis, there were considered also the usual TTT transform, the location independent riskier and the excess wealth orders. The first described transformation is a class which contains monotonic functions of random variables. The next were taken form the reliability theory: mixtures (with the emphasis on mixtures of exponential distributions) and formation of the coherent systems. The results in mixtures were used to compare lifetimes of coherent systems with involving the Samaniego signatures.

For the author the main results are:

- A) preservation theorems for the GTTT transform orders with respect to the distortion functions in comparison of
 - the GTTT transforms (theorems 3.12 i 3.14, pp. 29–30),
 - mixtures of exponential distributions (theorems 4.13, pp. 36);
- B) counterexamples to hypothesis that the likelihood ratio, the hazard rate and the reversed hazard rate orders are preserved under the formation of coherent systems (pp. 45–47), including overthrowing the theorem published by Navarro, Balakrishnan and Samaniego in *Journal of Applied Probability*, vol. 45, pp. 1097–1112.

Furthermore, for many other preservation rules proofs were conducted and examples were given.

The dissertation is dedicated to Professor Jarosław Bartoszewicz, whose scientific achievements inspired this research.