Abstract

Assume G is a group and $\mathcal{A}$ is an algebra of subsets of G closed under left translation. We study various ways to understand the Ellis group of the G-flow $S(\mathcal{A})$ (the Stone space of $\mathcal{A}$), with particular interest in the model-theoretic setting where G is definable in a first order structure M and $\mathcal{A}$ consists of externally definable subsets of G.

In one part of the thesis we explore strongly generic sets. Maximal algebras of such sets are shown to carry enough information to retrieve the Ellis group. A subset of G is strongly generic if each non-empty Boolean combination of its translates is generic. Trivial examples include what we call *periodic* sets, which are unions of cosets of finite index subgroups of G. We give several characterizations of strongly generic sets, in particular, we relate them to almost periodic points of the flow 2^G. For groups without a smallest finite index subgroup we show how to construct non-periodic strongly generic subsets in a systematic way. When G is definable in a model M, a definable, strongly generic subset of G will remain as such in any elementary extension of M only if it is strongly generic in G in an adequately uniform way. Sets satisfying this condition are called *uniformly strongly generic*. We analyse a few examples of these sets in different groups.

In the second part we assume that G is a topological group and consider a particular algebra of its subsets denoted $\mathcal{SBP}$. It consists of subsets of G that have the *strong Baire property*, meaning nowhere dense boundary. We explicitly describe the Ellis group of $S(\mathcal{A})$ for an arbitrary subalgebra $\mathcal{A}$ of $\mathcal{SBP}$ under varying assumptions on the group G, including the case when G is a compact topological group. We use this description to relate the Ellis groups computed for a model and its elementary extension in particular scenarios. Under some of those assumptions we also decide whether the obvious inclusions between the families of strongly generic, uniformly strongly generic and periodic sets can be reversed. These results can be applied in o-minimal structures, in which externally definable subsets are proved to have the strong Baire property. Finally, we propose a procedure of finding a maximal generic algebra in a given subalgebra of $\mathcal{SBP}$ given that we succeeded in doing so while neglecting nowhere dense sets.