

LÉVY MEASURES INVOLVING A GENERALIZED FORM OF  
FRACTIONAL INTEGRALS

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*Abstract:* A four-parameter fractional integral transformation  $\mathcal{A}_{\alpha,p}^{q,r}$  of measures on  $\mathbb{R}^d \setminus \{0\}$  is introduced and a systematic study of its properties depending on the values of the parameters is made. Descriptions of its domain, range, and effect on behaviors of measures near or far from the origin are found. A non-commutative relation with a two-parameter Upsilon transformation  $\Upsilon_{\beta,\theta}$  is established in the form  $\Upsilon_{\beta,\theta}\mathcal{A}_{\alpha,p}^{q,r} = \mathcal{A}_{\alpha,p}^{q,r}\Upsilon_{\beta',\theta'}$  for some  $\beta'$  and  $\theta'$ . Then the class of infinitely divisible distributions having Lévy measures of the form  $\mathcal{A}_{\alpha,p}^{q,r}\rho$  is discussed. It is represented as the class of laws of improper stochastic integrals with respect to Lévy processes if  $-\infty < \alpha < 1$ . For  $1 \leq \alpha < 2$ , it is the class of laws of essentially definable improper stochastic integrals.

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