EIGENVALUE DISTRIBUTION OF LARGE SAMPLE COVARIANCE MATRICES OF LINEAR PROCESSES

Oliver Pfaffel
Eckhard Schlemm

Abstract: We derive the distribution of the eigenvalues of a large sample covariance matrix when the data is dependent in time. More precisely, the dependence for each variable $i = 1, \ldots, p$ is modelled as a linear process

$$(X_{i,t})_{t=1,\ldots,n} = \left(\sum_{j=0}^{\infty} c_j Z_{i,t-j} \right)_{t=1,\ldots,n},$$

where $\{Z_{i,t}\}$ are assumed to be independent random variables with finite fourth moments. If the sample size n and the number of variables $p = p_n$ both converge to infinity such that $y = \lim_{n \to \infty} n/p_n > 0$, then the empirical spectral distribution of $p^{-1} \mathbf{X}\mathbf{X}^T$ converges to a non-random distribution which only depends on y and the spectral density of $(X_{1,t})_{t \in \mathbb{Z}}$. In particular, our results apply to (fractionally integrated) ARMA processes, which will be illustrated by some examples.

2000 AMS Mathematics Subject Classification: Primary: 15A52; Secondary: 62M10.

Keywords and phrases: Eigenvalue distribution, fractionally integrated ARMA process, limiting spectral distribution, linear process, random matrix theory, sample covariance matrix.

THE FULL TEXT IS AVAILABLE HERE