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THE QUANTUM DECOMPOSITION ASSOCIATED WITH THE LÉVY WHITE NOISE PROCESSES WITHOUT MOMENTS

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Abstract: The theory of one-mode type Interacting Fock Space (IFS) allows us to construct the quantum decomposition associated with stochastic processes on \mathbb{R} with moments of any order. The problem to extend this result to processes without moments of any order is still open but the Araki–Woods–Parthasarathy–Schmidt characterization of Lévy processes in terms of boson Fock spaces, canonically associated with the Lévy–Khintchine functions of these processes, provides a quantum decomposition for them which is based on boson creations, annihilation and preservation operators rather than on their IFS counterparts. In order to compare the two quantum decompositions in their common domain of application (i.e., the Lévy processes with moments of all orders) the first step is to give a precise formulation of the quantum decomposition for these processes and the analytical conditions of its validity. We show that these conditions distinguish three different notions of quantum decomposition of a Lévy process on \mathbb{R} according to the existence of second or only first moments, or no moments at all. For the last class a multiplicative renormalization procedure is needed.

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