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OPERATORS ON MARTINGALES, Φ-SUMMING OPERATORS, AND THE CONTRACTION PRINCIPLE

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Abstract: For the absolutely Φ-summing operators $T : X \to Y$ between Banach spaces X and Y we consider martingale inequalities of the type

$$\Big\| \sup_{1\le k\le N} \Big\| \sum_{l=1}^{k} T d_l \Big\|_Y \Big\|_{L_2} \le c \Big\| \sup_{i=1,2,\ldots} \Big(\sum_{k=1}^{N} |\langle d_k, a_i\rangle|^2 \Big)^{1/2} \Big\|_{L_2},$$

where $(d_k)_{k=0}^N \subset L_1^X(\Omega, \mathcal{F}, \mathbf{P})$ is a martingale difference sequence and $(a_i)_{i=1}^\infty$ is a sequence of normalized functionals on X, and we show that these inequalities are useful in different directions. For example, for a Banach space $X, x_1, \ldots, x_n \in X$, independent standard Gaussian variables $g_1, \ldots, g_n$, and $1 \le r < \infty$ we deduce that

$$\Big\| \sum_{i=1}^{n} \Big[\sum_{k=\tau_{i-1}+1}^{\tau_i} d_k \Big] x_i \Big\|_{L_r^X} \le c\sqrt{r} \Big\| \sup_{1\le i\le n} S_2({}^{\tau_{i-1}}f^{\tau_i}) \Big\|_{L_r} \Big\| \sum_{i=1}^{n} g_i x_i \Big\|_{L_1^X},$$

where $f = (d_k)_{k=0}^N$ is a scalar-valued martingale difference sequence such that $(|d_k|)_{k=1}^N$ is predictable, $0 = \tau_0 \le \tau_1 \le \ldots \le \tau_n = N$ is a sequence of stopping times, and

$$S_2({}^{\tau_{i-1}}f^{\tau_i}) := \Big(\sum_{k=\tau_{i-1}+1}^{\tau_i} |d_k|^2 \Big)^{1/2}.$$

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