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MONOTONICITY AND NON-MONOTONICITY OF DOMAINS OF STOCHASTIC INTEGRAL OPERATORS

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Abstract: A Lévy process on $\mathbb{R}^d$ with distribution μ at time 1 is denoted by $X^{(\mu)} = \{X_t^{(\mu)}\}$. If the improper stochastic integral $\int_0^{\infty-} f(s)dX_s^{(\mu)}$ of f with respect to $X^{(\mu)}$ is definable, its distribution is denoted by $\Phi_f(\mu)$. The class of all infinitely divisible distributions μ on $\mathbb{R}^d$ such that $\Phi_f(\mu)$ is definable is denoted by $\mathcal{D}(\Phi_f)$. The class $\mathcal{D}(\Phi_f)$, its two extensions $\mathcal{D}_c(\Phi_f)$ and $\mathcal{D}_{es}(\Phi_f)$ (compensated and essential), and its restriction $\mathcal{D}^0(\Phi_f)$ (absolutely definable) are studied. It is shown that $\mathcal{D}_{es}(\Phi_f)$ is monotonic with respect to f, which means that $|f_2| \le |f_1|$ implies $\mathcal{D}_{es}(\Phi_{f_1}) \subset \mathcal{D}(\Phi_{f_2})$. Further, $\mathcal{D}^0(\Phi_f)$ is monotonic with respect to f but neither $\mathcal{D}(\Phi_f)$ nor $\mathcal{D}_c(\Phi_f)$ is monotonic with respect to f. Furthermore, there exist μ, f_1 and f_2 such that $0 \le f_2 \le f_1$, $\mu \in \mathcal{D}(\Phi_{f_1})$, and $\mu \notin \mathcal{D}(\Phi_{f_2})$. An explicit example for this is related to some properties of a class of martingale Lévy processes.

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