Imaginaries in pseudo-*p*-adically closed fields

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Proposition (Montenegro)

Let K be a bounded pseudo-p-adically closed field. There are finitely many p-adic valuations on K and they are definable in the ring language.

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- The *geometric language* $\mathcal{L}^{\mathcal{G}}$ has sorts $\mathbf{F}$, $\mathbf{S}_n$ and $\mathbf{T}_n$ for all $n \geq 1$. It also contains the ring language on $\mathbf{F}$, the canonical projections $s_n : \mathrm{GL}_n(\mathbf{F}) \to \mathbf{S}_n$ and $t_n : \mathrm{GL}_n(\mathbf{F}) \to \mathbf{T}_n$.

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Proposition

Let $K_0 \subseteq A \subseteq \mathbf{F}(M)$ and $s_i, t_i \in \mathbf{S}_{i,n}(M)$. If

$$\forall i,\ s_i \equiv^{M_i}_{\mathcal{L}_i(A)} t_i$$

then

$$(s_i)_{i\leq n} \equiv^{M}_{\mathcal{L}(A)} (t_i)_{i\leq n}.$$

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Let $c \in \mathbf{F}(M)$ and $d \in \mathcal{G}_i(\mathrm{acl}^{\mathrm{eq}}_M(Ac))$ be some tuples. Assume $\mathrm{tp}^{\overline{M}_i}_{\mathcal{L}_i}(c/\overline{M}_i)$ is $\mathcal{G}_i(A)$-invariant, then so is $\mathrm{tp}^{\overline{M}_i}_{\mathcal{L}_i}(d/\overline{M}_i)$.

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Let $c \in \mathbf{F}(M)$ be some tuple. Then, for all i, there exists a $\mathcal{G}_i(A)$-invariant $\mathcal{L}_i(\overline{M}_i)$-type p_i such that $\mathrm{tp}^M_{\mathcal{L}}(c/A) \cup \bigcup_i p_i$ is consistent.

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Corollary

Let $c \in \mathbf{F}(M)$ be some tuple. Then, for all i, there exists a $\mathcal{G}_i(A)$-invariant $\mathcal{L}_i(M_i)$-type p_i such that $\mathrm{tp}^M_{\mathcal{L}}(c/A) \cup \bigcup_i p_i$ is consistent.

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- For all $C \leq M^{\mathrm{eq}}$ and all tuples $a, b \in \mathcal{D}(M)$, write $a \downarrow_C b$ if there are $\mathcal{R}_i(A)$-invariant $\mathcal{L}_i(M_i)$-types p_i with $a \vDash \bigcup_i p_i|_{\mathcal{R}_i(C)b}$.

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2. For all $A = \mathrm{acl}_M(A) \subseteq M$ and $a, b, c \in \mathcal{D}(M)$ tuples, if $b \downarrow_A a$, $c \downarrow_A ab$, $a \equiv^M_{\mathcal{L}(A)} b$ and $ac \equiv^{M_i}_{\mathcal{L}_i(\mathcal{R}_i(A))} bc$, for all i, then there exists d such that $db \equiv^M_{\mathcal{L}(A)} da \equiv^M_{\mathcal{L}(A)} ca$.

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- If $A \subseteq \mathbf{F}$, this is an earlier result of Montenegro.
- The general result follows from the older version and the description of the structure on the geometric sorts given by the orthogonality result.

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- Since the valuations v_i are discrete, $\mathbf{T}_{i,n}$ is coded in $\mathbf{S}_{i,n+1}$.
- Let $\mathcal{O} = \bigcap_i \mathcal{O}_i$. We have a bijection

$$\prod_i \mathbf{S}_{i,n} = \prod_i \mathrm{GL}_n(\mathbf{F})/\mathrm{GL}_n(\mathcal{O}_i) \simeq \mathrm{GL}_n(\mathbf{F})/\mathrm{GL}_n(\mathcal{O}).$$