

Game theory

Problem list 1

For each of the games described below, define the strategy sets of the players, the payoff function (or matrix), and the game values for each of the players. Assume that Problems 6–10 describe two player zero-sum games.

1. The participants of this multiplayer game consider taking a certain initiative (action). Each of them can make one of two decisions: “take part in the action” or “abstain”. If more than half of the participants take part in the action, then each of them receives a payoff of 100. If the participants who decide to take part do not reach the majority, then each of them pays a “fine” of 10. In either case, the players who do not join in do not receive payoffs and do not pay fines.
2. Two suspects are being interrogated simultaneously in separate rooms. Each of them can either confess to the alleged crime or deny the charges. If both confess, then they will be sentenced to 5 months in prison each; if both deny the charges, then each will be sentenced to 2 months. If one confesses (implicating the other) and the other one doesn’t, then the one who confesses will receive a lighter sentence (1 month), whereas the other one will be treated more harshly (10 months in prison).
3. Two workers perform a task, and each of them can either work (then $s_i = 1$) or pretend to work (then $s_i = 0$), for $i = 1, 2$. If a player works, they incur a cost of 3, and if they don’t, they don’t incur any cost. The value of the task performed is equal to $2(s_1 + s_2)$ to each of the players, regardless of whether they worked or merely pretended to.
4. Two players are driving home and reach a road blocked by an avalanche. The total energy required to clear the snow off the road is equal to $c > 0$, and the benefit each reaches for arriving home is $b > c$. The energy (payoff) for each of the players, when neither of them does anything, is equal to $a < b - c$.
5. Two players have an amount 100 to share between themselves. Player A suggests to player B splitting the amount as follows: x for B and the rest for A , for some $x \in \{1, 2, \dots, 100\}$. Player B can either agree (then the payoffs will be as suggested by A) or disagree (then both will receive 0).
6. Three identical sets are given: $L_I = L_{II} = L = \{1, 2, 3, 4\}$. Player I chooses a number x from L_I , and player II chooses a number y from L_{II} . Then a number Q is chosen at random from L . After revealing the numbers x , y and Q , a prize of 100 is awarded to player I if one of the inequalities

$$y < x < Q, \quad Q < x < y$$

is satisfied. The same prize is awarded to player II if one of the inequalities

$$x < y < Q, \quad Q < y < x$$

is satisfied.

7. From a pile that initially has 4 coins, two players take either 1 or 2 coins in turns. Player I starts, and the one who finishes the game (i.e. takes the last coin) receives an amount of 10 from the other player.

8. Player I writes one of the 26 letters of the alphabet on a card. Without showing the card to player II, they tell the letter they wrote on the card, but they can lie. Player II decides whether or not player I was telling the truth, guessing *true* or *false*. If player I is caught lying by player II, then player I pays an amount of 100, whereas if player II guesses correctly that player I was telling the truth, then player I pays 20. If player II guesses incorrectly, then player I receives 60.
9. In a simplified version of the Italian game *Morra*, each player shows either one or two fingers and simultaneously guesses how many fingers are shown by the opponent. If both guess correctly or both guess incorrectly, then the payoff is equal to zero. Otherwise, the player who guessed correctly receives a payoff equal to the total number of fingers shown by both of the players.
10. Consider the game of drawing cards from the hat (discussed in the lectures) assuming that player I does not know the card they have randomly drawn.