

Game theory

Problem list 3

1. Among cards labelled 1, 2 and 3, two cards are selected at random and handed to two players, so that each known only their own card. The players simultaneously make a declaration, saying *higher* or *lower*. If both declare *lower*, the cards are compared and the player with a higher-numbered card receives 1 from the opponent. If both declare *higher*, the cards are also compared, but this time the player with a higher-numbered cards receives 2 from the opponent. If the declarations differ, then the player who declared *lower* can either pass, paying 1 to the opponent, or request a comparison, in which case the payoffs are the same as if both declared *higher*.

For this random game, define the sets of actions and strategies, and compute the payoff matrix.

2. Two teenagers are driving in front of each other on a single-lane road, using their parents' cars. Just before the collision, each of them can either swerve (*c*) or continue going straight (*d*). If both swerve, then the payoffs are $(0, 0)$, whereas if one swerves and the other one does not, then the payoffs are $(0, R)$ or $(R, 0)$. If they collide, then the payoffs are $(R/2 - K, R/2 - K)$, where K is the penalty received by each of the teenagers. Assume that the decisions of the players are influenced by the type of their parents – harsh (*H*) or lenient (*L*) – and that Nature “assigns” the types for each pair of parents uniformly at random (with probability $1/4$ for each possibility). If the type of a parent is *H*, then $K = H$, whereas if it is *L*, then $K = L < H$.

For this game, define the sets of actions and strategies, and using a game tree compute the payoff matrix.

3. Find NE points for the two player game with incomplete data, defined as follows:

- sets of player types: $\Theta_1 = \{t\}$, $\Theta_2 = \{t_1, t_2\}$;
- sets of actions: $A_1 = \{a, b\}$, $A_2 = \{c, d\}$;
- convictions of players: $\Phi_1(t_1|t) = 0.6$, $\Phi_2(t|t_i) = 1$ for $i = 1, 2$;
- payoff matrices for types t_1 and t_2 (respectively) of the second player:

$$\begin{pmatrix} (1, 2) & (0, 1) \\ (0, 4) & (1, 3) \end{pmatrix}, \quad \begin{pmatrix} (1, 3) & (0, 4) \\ (0, 1) & (1, 2) \end{pmatrix}.$$

4. A farmer has to choose one of three possible dates for sowing crops: I, II or III. The yield depends on the future weather (four possible states: A, B, C or D) and the date of sowing as follows:

	A	B	C	D
I	21	15	32	16
II	28	20	10	20
III	13	27	25	15

Determine which date for sowing the farmer will choose, if they:

- (a) don't know the frequency of each state of the weather;
- (b) are a moderate optimist, i.e. their optimism coefficient is equal to 0.7;
- (c) are trying to minimise loss resulting from taking a wrong decision.